3 An overview of CGE models

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3.1 An introduction to Computable General Equilibrium Models

3.1.1 Introduction

Computable General Equilibrium (CGE) models seek to understand the effect of external shocks in the economy through the simultaneous equilibrium that occurs in the markets. Its main advantage lies in its capacity to comprise the linkages among all sectors. Thus, whenever a sector is shocked by any policy or market situation, then CGE models can estimate a new equilibrium that is produced simultaneously, in all sectors. It provides the expected changes in prices and quantities across all sectors. Moreover, from these new equilibria, all kinds of macroeconomic and microeconomic indicators may be built, such as Gross Domestic Product (GDP), employment, or equivalent variation measures.

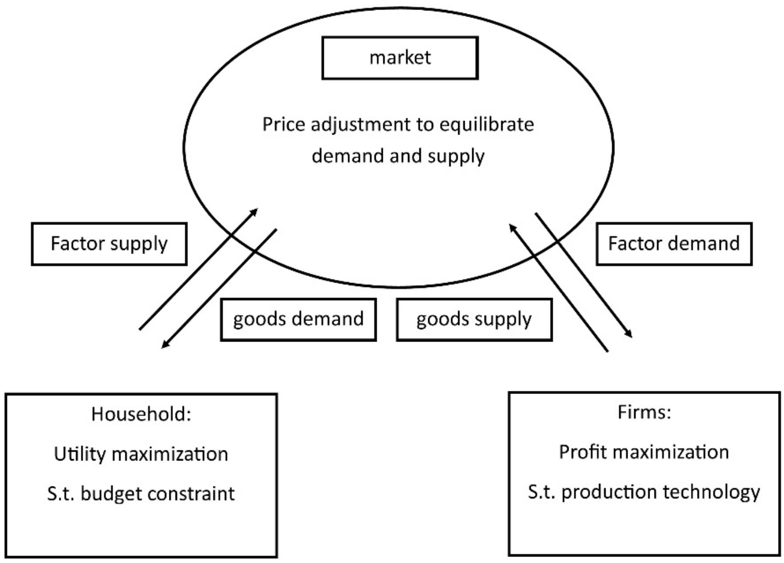
CGE models are based on statistical datasets such as Input-Output Tables (IOT), Social Accounting Matrices (SAM) or Satellite Accounts. Thus, the models are built upon a solid basis of real datasets. However, CGE requires further assumptions to estimate the models. More precisely, the modeler needs to define the functional forms chosen for production, demand or supply, as well as the elasticities behind such functions. Moreover, the modeler also needs to define a model closure to be able to estimate the model. Such decisions condition the results obtained; it is important to anticipate the way this may occur and proceed accordingly.

This paper provides an overview of the theoretical underpinnings of the CGE model as well as other relevant impact assessment models. It focuses on the market closure implications to anticipate its relevance for project appraisal and any divergence with respect to Cost-Benefit Analysis (CBA).

3.1.2 The foundations of the CGE model

The theoretical foundation of CGE models was established by Arrow and Debreu (1954). Paraphrasing Böhringer, Rutherford and Wiegard (2003), “CGE models combine microeconomic theory (general equilibrium theory) with data sets (SAM) in order to derive policy insights". While the model has traditionally been regarded as a *black-box* because of its complexity, it has the advantage of being able to deal with the whole economy by following the circular flow of income and expenditure (Wing, 2004) (see Figure 1).

Figure 1. Economic structure of a CGE model



Source: Hosoe, Gasawa and Hashimoto (2010)

A basic CGE model implies the existence of a representative household[1] who owns the factors of production (labour and capital). These factors are demanded by firms to produce goods and services that are demanded by the representative household and by the rest of the economy's sectors as inputs. Finally, the representative household demands goods and services constrained by their income (rent from factors of production). Hence, by developing these economic interactions, the change that takes place in one sector or economic agent causes an economic change in other sectors affecting the prices, quantities, or incomes of the economy. Thus, it not only account for *direct effects*, but also *indirect and multiplier effects*.

[1] The existence of a representative household allows welfare analysis to be conducted in CGE through the calculus of the equivalent variation (See Hosoe, Gasawa & Hashimoto, 2010).

This basic structure can be extended to include other economic agents (additional representative households or the government), the investment, or imports and exports (open economy). However, all these extensions must work under the circular flow of income. For instance, if an additional representative household is considered, then the model must specify its endowment of factors and the kinds of goods demanded. Moreover, it should be noted that the total supply of factors of both households must be demanded by firms, and that the goods of the economy demanded by both agents must be produced by the firms (or imports) of the economy. Finally, both households decide their demand of goods constrained to their disposable income (rent of factors).

Mathematically, the circular flow of income and expenditure can be summarized by the following three conditions: zero benefit, market clearance conditions, and income balance (Böhringer, et al, 2003; Hosoe, et al, 2010). Following the notation considered by Böhringer, et al, 2003, the three of them form the so-called Walrasian equilibrium, i.e. prices and quantities vary simultaneously so as to fulfill the following three economic conditions:

3.1.3 Zero benefit condition

Firms supply goods and services to the market. In order to do so, they combine capital, labour and intermediate goods to produce. In this process, the firms pay wages to workers, rents to capital owners and intermediate demand to other firms. The value of inputs per activity must be equal to or greater than the value of outputs.

$$\pi_j(p) = R_j(p) - C_j(p) \geq 0 \quad \forall j$$

where $\pi_j(p)$ represents the benefit by activity j, $R_j(p)$ and $C_j(p)$ are the unit cost functions and unit revenues functions by activity j, respectively, and p is a non-negative vector of prices for all goods and factors.

$$C_j(p) \equiv min\left\{\sum_i p_i \frac{\partial \pi_j(p)}{\partial p_i} \mid f_j(.) = 1\right\}$$

$$R_j(p) \equiv max\left\{\sum_i p_i \frac{\partial \pi_j(p)}{\partial p_i} \mid g_j(.) = 1\right\}$$

3.1.4 Market clearance conditions

The production generated in the zero-benefit condition is supplied to the market to be purchased as final demand (household consumption, government consumption, investment and exports); or as intermediate demand to produce other goods and services by the firms. The supply of any commodity must equal or exceed consumers' demand:

$$\sum_j y_j \frac{\partial \pi_j(p)}{\partial p_i} + \sum_h w_{i,h} \geq \sum_h d_{i,h}(p, M_h)$$

where y_j represents the supply of good by activity j. $\sum_h w_{i,h}$ represents the initial endowment of good i by institution h. $\sum_h d_{i,h}(p, M_h)$ represents the final demand for good i by institution h given prices p and income M. $d_{i,h}(p, M_h)$ stands for the final demand obtained from the maximization problem of the representative household:

$$d_{i,h}(p, M_h) \equiv argmax\left\{U_h(x) | \sum_i p_i q_i = M_h\right\}$$

Finally, $U_h(x)$ denotes the utility function of household h.

3.1.5 Income balance conditions

Households are endowed with income obtained from firms as workers and capital owners. The households employ this income to demand goods and services as well as investment. The income (value of the endowment) of each institution (households, mainly) h must be equal or exceed the final demand, so that:

$$\sum_i p_i w_{i,h} = M_h \geq \sum_i p_i d_{i,h}$$

$\sum_i p_i w_{i,h}$ represents the value of the endowment for institutions h, and $\sum_i p_i d_{i,h}$ represents the value of the final demand of institutions h.

The aforementioned conditions provide a consistent framework for the economic analysis of policies with sectoral changes, linkage effects and welfare evaluation. These equations need to be calibrated according to a SAM (see section 3.2.2) to replicate an initial equilibrium (see applications in Appendix 1). Finally, these conditions must be complemented with the *model/ macro closure*[2]. Basically, from a modelling perspective, the macro closure ends up assuming which variables are endogenous or exogenous (Hosoe, et al, 2010). In this regard, there are three key variables or decisions to be made when closing the model: investment, government and current account (open economy). Such assumptions have economic consequences and yield different results. For instance, for a closed economy without government, the following identity holds, $S = I$. In these circumstances, savings *(S)* or investment *(I)* must be fixed or a new equation has to be included to determine their respective values. If the investment is fixed, savings will adjust freely (investment-driven or Johansen closure). On the other hand, if savings remain fixed, this model follows a savings-driven closure.

The same reasoning can be considered when addressing the government and current account closure. For instance, some governments may face a binding budget restriction *(D)*. In this case, it is reasonable to assume a fixed budget where expenditure and income vary in consequence. Finally, the current account closure implies determining savings, investments, or the current account. In general, most CGE models assume a fixed current account while they opt for a savings-driven or an investment-driven closure. This closure is common for small economies where foreign credit may be limited (Gilbert and Tower, 2013). In any case, there is no ideal macro-closure, as it relies on the kind of policy simulation carried out.

The structure of a CGE model can be relaxed or the SAM enriched to address different issues such as externalities, non-market goods or obtain a higher sectoral disaggregation. In this sense, the inclusion of natural resources in traditional IOT has allowed the widespread development of environmental analysis using CGE models (Bergman, 2005; or Britz and Hertel, 2011). For instance, CGE models have been especially fruitful when modeling a CO2 emissions trading scheme (Böhringer, 2002: or Wing, 2006). SAM can also be expanded to deal with several economies, known as

[2] The model closure is specifically addressed in section 3.4.

multi-regional models (Aguiar, Narayanan and McDougall, 2016) and spatial markets interactions (Mercenier et al., 2016).

The main assumptions of a standard CGE model can be summarized as follows:

- Circular flow of income and expenditure.
- Secondary production allowed.
- A minimum of one representative household.
- Non-capacity constraints.
- Constant return to scale.
- Perfect market competition.

As said, the latter four assumptions can be relaxed to tackle more than one representative household, include capacity constraints, increasing or decreasing returns to scale, or imperfect market competition (unemployment, monopoly, or oligopoly market behaviour) (Roson, 2006; Boeters and Van Leeuwen, 2010; or Boeters and Savard, 2011). Moreover, the behaviour of consumer and firms can be modeled according to four different kinds of function:

- Leontief (elasticity of substitution equals zero)
- Cobb-Douglas (elasticity of substitution equals 1)
- Constant elasticity of substitution (CES) (elasticity of substitution different from 1)
- Stone-Geary (elasticity of substitution different from 1)

The latter also allows for income elasticity different from 1, but at the cost of generating non-homothetic preferences.

On the other hand, there are two main approaches when programming a CGE model: i) maximizing representative household utility where the remaining conditions operate as constraints (Hosoe, et al, 2010; or Gilbert and Tower, 2013) or ii) solving the problem as a system of equations where variables and equations form a Mixed Complementarity Problem (MCP), by avoiding any maximizing behaviour (Böhringer, et al, 2003) (see applications in Appendix 1). Further, Rutherford (1999) developed a straightforward subsystem (MPSGE) to program CGE models in MCP syntax[3]) (see applications in Appendix 1).

3.2 An overview of other impact assessment models

3.2.1 Input-Output tables

Input-Output Analysis (IOA) is a methodology that precedes the CGE model, and was first developed by Leontief (1936, 1941). The methodology can quantify the economic impact of economic policies, events, or projects in the whole economy by assuming exogenous changes in the final demand, taxes, or subsidies (Miller and Blair, 2009). Similar to CGE, the IO methodology relies on the same rationale of the interdependences of the economic sectors of an economy (*economic linkages*), where the production of any sector is demanded as inputs by other sectors to produce their own goods, and so on. Hence, they are also capable of capturing direct, indirect, and multiplier effects.

However, IOA cannot tackle simultaneous changes in prices and quantities, as done in CGE. In fact, the methodology can only distinguish between the demand and price model. Traditional IOA can be characterized by the following and more restrictive assumptions:

- Leontief production technology (fixed proportions).
- Constant returns to scale.
- No secondary production.
- Non-capacity constraint.
- One single household.

However, some of these assumptions can be relaxed to encompass more realistic economic behaviour. For instance, Miyazawa (2012) provides a comprehensive explanation by including more than one representative consumer in an IO framework. On the other hand, Raa (2006) analyzes the inclusion of secondary production and

[3] See Markusen (1995) for self-study examples in a Mathematical Programming System for General Equilibrium Analysis (MPSGE).

Cobb-Douglas production technologies, while explaining the conceptual boundaries for the inclusion of increasing economies to scale in IOA. According to Raa (2006), IO models can be translated into linear programming, allowing the introduction of capacity constraints in the production system to be addressed. Most of the current developments in IOA have been focused on dealing with environmental aspects (Miller and Blair, 2009; Wiedmann, Minx, Barrett and Wackernagel, 2006; Wiedmann, 2009; Raa, 2006; or Lenzen, 1998). In sum, IOA is also equipped to quantify the economic impact of an economic policy on the economy, as done in CGE. Nevertheless, it is based on more restrictive assumptions.

Input-Output Tables (IOT) form the main dataset to develop a CGE model because they follow the circular flow of income. These tables are usually elaborated by the Office for National Statistics and are publicly available. They are a natural extension of the national accounts (the production and consumption accounts) and emphasize intersectoral relationships. The national accounts follow standard international procedures for their development and international comparison (SNA, 1993). Three main blocks can be distinguished in IOT:

1. Intermediate demand block (intersectoral/inputs demand).
2. Final demand block (household consumption, government consumption, investment, and exports by goods).
3. Primary inputs block (remuneration of labour and capital and employees by sectors).

Table 1 shows the general structure of an IOT, the intermediate demand block, with the sectors in rows and columns, representing the intermediate demand ($id_{i,j}$) of each sector, i.e., the production of each sector that is demanded by the others to produce their goods. The final demand column represents the share of the sectoral production that is demanded for consumption (representative household and the government), investments, or exports. Finally, the total demand by goods (intermediate and final demand) equates the total production by sector ($\sum_{j=1}^{n} id_{i,j} + fd_i = X_i$). Similarly, the total sectoral production (X_i) equates the value of the factors demanded as inputs

($\sum_{j=1}^{n} id_{j,i} + salaries_i + c\_capital_i = X_i$) ensuring that the circular flow of income holds. Finally, the last row includes the number of employees by sector.

Table 1. A simplified Input-Output Table

	$\boldsymbol{Sector_1}$	....	$\boldsymbol{Sector_n}$	***Final demand***	***Total demand***
$\boldsymbol{Sector_1}$	$id_{1,1}$	....	$id_{1,n}$	fd_1	$\sum_{j=1}^{n} id_{1,j} + fd_1$
....	....	....	....	....	....
$\boldsymbol{Sector_n}$	$id_{n,1}$	....	$id_{n,n}$	fd_n	$\sum_{j=1}^{n} id_{n,j} + fd_n$
salaries	$salaries_1$	....	$salaries_n$		
cost of capital	$c\_capital_1$	....	$c\_capital_n$		
Total production	x_1	....	x_n		
Employment	L_1	....	L_n		

3.2.2 Social Accounting Matrix (SAM)

The IOT provide detailed information about the intersectoral relationships of an economy, the source of the production (supply: domestic and imported) and its respective destination (intermediate demand or final demand). However, they lack a more comprehensive characterization of the households and/or government (Miller and Blair, 2009). The SAM bridges this gap by including transfers among institutions, social transfers and direct taxation to households and firms; as well as the relationship of all of them with the rest of the world (household account, value-added account, capital accumulation account, the balance of payments account and the government account). Hence, the SAM enrich or complement the IOT by characterizing the successive income distributions that take place in the economic system (Breisinger, Thomas and Thurlow, 2009). Table 2 shows the structure of a standard SAM. The IOT is highlighted in blue, while the remaining accounts that form the SAM are in red. The IOA can be easily extended to take into account this new information and develop their respective multipliers (Miller and Blair, 2009; Breisinger, et al, 2009).

Table 2. Standard Social Accounting Matrix

	Activities	**Commodities**	**Factors**	**Households**	**Government**	**Savings and investment**	**Rest of the world**	**Total**
Activities		Domestic supply						Activity income
Commodities	Intermediate demand			Consumption spending	Recurrent spending	Investment demand	Exports	Total demand
Factors	Value-added							Total factor income
Households			Factor payment to households		Social transfers		Foreign remittances	Total household income
Government		Sales taxes and import tariffs		Direct taxes			Foreign grants and loans	Government income
Savings and investment				Private savings	Fiscal surplus		Current account balance	Total savings
Rest of the world		Imports						Foreign exchange outflow
Total	Gross output	Total supply	Total factor spending	Total household spending	Government expenditure	Total investment spending	Foreign exchange inflow	

Source: Adapted from Breisinger, *et al*. (2009)

The SAM focuses firstly on the primary factor incomes generated in the economic process (compensation of employees, gross operating surplus or indirect taxes) that must be assigned to different economic agents (households, firms, or government). But these agents can be resident or non-resident. At the same time, resident agents can also receive income from abroad.

The secondary income process predominantly comprises the government. The government collects the money required for public spending through direct and indirect taxation but also for paying subsidies and any other social provision. The role of the government generates a second income distribution that allows calculation of the gross national disposable income. At the same time, the latter can be disentangled into final consumption and savings.

Finally, and briefly, the domestic economy exchanges not only goods and services (imports and exports) or rents, but assets with and from abroad. This economic activity is registered in the accumulation account (capital and financial accounts). The inclusion of the aforementioned aspects together with the IOT comprise the SAM. Obviously, the SAM provides a richer set of information about economic relations than the IOT.

3.2.3 Satellite accounts

Satellite accounts deal with activities that are insufficiently covered by the standard national accounts. For instance, satellite accounts have been built for tourism (TSA, 2008; Frechtling, 1999 and 2010), culture (FCS, 2009; Throsby, 2008) and the environment (SEAA, 2012; Muller, Mendelsohn and Nordhaus, 2011; Bartelmus, Stahmer and Tongeren, 1991), among others. They quantify the direct contribution of the corresponding activities into the economy in terms of employment and sectoral production, demand or GDP. The information is usually deployed in a set of tables organized by topics. International organizations such as the United Nations (UN) and the Organization for Economic Co-operation and Development (OECD), in cooperation with other institutions, develop and release methodological frameworks for constructing comparable and harmonized satellite accounts, which will be implemented by the respective National Statistical Office. For instance, the Tourism Satellite Account has been conceived to distinguish consumption incurred by residents and tourists. This is the only way to understand the role of tourists in a multisector framework such as the IOT. More precisely, it disentangles the production by goods into tourism and non-tourism activities and distributes total non-resident consumption into the different goods categories of the IOT (Inchausti-Sintes, 2015). On the one hand, the account can be directly employed to quantify the economic contribution of tourism into the economy (Bryan, Jones and Munday, 2006) or to understand the contribution of any tourism subsector, such as maritime tourism (Diakomihalis, 2007). Moreover, both tourism and environmental accounts can be combined to shed light on the consequences of tourism activities in the environment (Collins, Jones and Munday, 2009). On the other hand, the environmental account has been mostly developed to extend the IOT (Liang, et al, 2017), generating the so-called energy environmental IOT (Burniaux and Truong, 2002).

3.3 Further extensions to the CGE models

3.3.1 Dynamic Computable General Equilibrium

A SAM provides a snapshot of an economy in one period. Nevertheless, many economic policies take place over several periods/years. CGE models can be adapted to evolve over time. Such adaptation implies the following variables and parameters and their respective assumptions: economic growth (g), capital depreciation (δ), interest rate (r) and the initial stock of capital (K_0). These parameters and variables have to be set according to certain equations in order to assure a steady state economic growth, i.e. that the circular flow of income and expenditure holds over time. Following Paltsev (2008), a dynamic CGE model can be introduced as follows:

The initial stock of capital must equal the capital earnings (gross operating surplus, VK) divided by the initial return to capital ($\delta + r$).

$$K_0 = \frac{VK}{\delta + r}$$

At the same time, the stock of capital multiplied by ($\delta + g$) must equal the initial investment level (I_0). In general, the initial investment level is obtained from the IOT:

$$I_0 = (\delta + g)K_0$$

The stock of capital will evolve according to the following equation:

$$K_{t+1} = K_t(1 - \delta)K_t + I_t$$

where I_t represents the investment level in period t. The remaining conditions of a standard CGE model holds in each period. Finally, the general structure of a dynamic model is as follows:

$$max \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^{t} U(C_t)$$

$s.t.$:

$$C_t = F(K_t, L_t) - I_t$$

$$K_{t+1} = K_t(1-\delta)K_t + I_t$$

where the objective function denotes the present value of the utility ($U(C_t)$) of the representative household, ρ represents the individual time-preference, C_t refers to total consumption, and $F(K_t, L_t)$ represents total production.

A last key assumption concerns the behaviour of the representative households. Depending on the kind of assumption, dynamic CGE models[4] can be split into forward-looking (Ramsey, 1928) and backward-looking models (or recursive-dynamic models). The main difference between them is their representation of future expectation. In the former, agents/households have perfect expectations, whereas in the latter, they form their expectations in the decision-making moment. Forward-looking models imply deeper changes in the economic structure than backward-looking models (Babiker, Gurgel, Paltsev and Reilly, 2009).

3.3.2 Dynamic Stochastic Computable General Equilibrium (DSGE)

Dynamic CGE models can also encompass stochastic analysis. Although they can be regarded as an extension to traditional CGE models, they have followed a different theoretical and applied approach more focused on macroeconomic analysis (Wickens, 2011; Junior, 2016; or Walsh, 2017). Briefly, the advent of DSGE can be traced back to the Real Business Cycle model (RBC) developed by Kydland and Prescott (1982). This model formalized the macroeconomic process according to maximizing and

[4] See Dixon and Rimmer (2010) or Fougére, Mercenier and Mérette (2007) for applications of dynamic CGE models.

minimizing behaviours, first order conditions and rational expectations, instead of *ad hoc* aggregated macroeconomic models. However, the widespread development and application of DSGE occurred when frictions were included in the model, which allowed for more realistic economic situations, in so-called New Keynesian models. In this sense, DSGE models can encompass complex economic behaviour such as sticky prices and salaries (Smets and Wouters, 2003), risk premium (Adolfson, Laséen, Lindé and Villani, 2007), dollarization (Castillo, Montoro and Tuesta, 2013), or policy analysis (Del Negro and Schorfheide, 2013; or Hohberger, Priftis, and Vogel, 2020), among other topics.

However, the calibration procedure in stochastic models follows a more complex approach where the rank conditions and thus, the initial solution, are not always achieved. Most authors in applied studies opt to work in logarithms (*log-linearization*) to reduce complexity between economic variables (highly non-linear models) and more easily achieve a mathematical solution (DeJong and Dave, 2011). These mathematical difficulties are also explained by the inclusion of rational expectations (forward or backward-looking), affecting the eigenvalues of the model.

On the other hand, while traditional static and dynamic models assume the parameters of the model as given, a stochastic approach can estimate these parameters econometrically by including time series data. Briefly, the parameters can be estimated following two main approaches: the Kalman filter and a Bayesian estimation (Fernández-Villaverde, Rubio-Ramirez and Schorfheide, 2016). The former implies working in a state-space framework, while the latter requires assuming distribution functions for the parameters. In both cases, once the initial conditions are fulfilled, the algorithm allows for a quick and reliable convergence. This is especially useful when dealing with short aggregated macroeconomic series.

While the aforementioned limitations of stochastic CGE models affect their applicability in project evaluation, they have been widely applied in macroeconomics. In this sense, they have become a key tool for central banks to conduct macroeconomic forecasting and/or monetary policy analysis (Smets and Wouters, 2004; or Tovar, 2009). From an academic perspective, in contrast to traditional macroeconomic models, a stochastic CGE approach provides a robust theoretical microeconomic foundation, and allows for econometric testing of economic theories.

3.4 The implications of alternative model closures in CGE

3.4.1 The concept of model closure

As stated by Gilbert and Tower (2013), a model is mathematically “closed” when we have enough independent equations to explain the endogenous variables. Further, the selection of exogenous and endogenous variables also determines the computability and complexity of the model (Hosoe, et al, 2010). As noticed by Decaluwé and Monette (1988), Sen (1963) was one of the first who shed light on this issue by showing the complexity of simultaneously determined several economic variables in one single model. Nevertheless, from an economic perspective, the distinction between exogenous and endogenous variables goes beyond its mathematical tractability. The model closure directly alters the economic adjustment of the model and hence, the policy conclusions (Taylor and Lysy, 1979). Specifically, it affects key aspects of a project, such as its financing, which can be done through direct taxation, indirect taxation, private savings, or debt raised from international capital markets. Each aspect has different implications for income redistribution and the economy’s future dynamics.

Model closure is relevant because it affects the social welfare measures taken in a CGE model. To date, the literature on CGE has addressed the issue of model closure focusing on its macroeconomic impact and sectoral implications, rather than its effect on welfare (Sen, 1963; Decaluwe and Monette, 1988; Dewatripont and Michel, 1987; Rattsø, 1982; Robinson, 2006; Adelman and Robinson, 1988; Doi, 2006; Hosoe, et al, 2010 or Gilbert and Tower, 2013). In some of these cases, they do not even explicitly model a representative household - the so-called macro CGE models (Sen, 1963; Dewatripont and Michel, 1987; Rattsø, 1982; Robinson, 2006) - but assume non-homothetic preferences in household behaviour, which impede welfare comparisons (Adelman and Robinson, 1988). In other cases, they describe the theoretical macroeconomic implications of adopting some of the closures (Doi, 2006; Hosoe et al, 2010 or Gilbert and Tower, 2013).

Thissen (1998) briefly introduces Sen´s model mathematically as follows:

$$Y = f(K, L) \qquad (1.a)$$

$$w = \frac{\partial f(K,L)}{\partial L} \qquad (2.a)$$

$$Y = rK + wL \qquad (3.a)$$

$$S = S_p rK + S_w wL \qquad (4.a)$$

$$I = I^* \qquad (5.a)$$

$$S = I \qquad (6.a)$$

Equation (1.a) denotes the production function where K denotes capital, and L denotes labour, which comprise the factors of production. Equation (2.a) represents the demand of labour from production Y, equation (3.a) denotes the income constraint of this economy where income depends on rents from capital (rK) and labour (wL), with r and w representing the rent of capital and wage, respectively. The income constraint also equates total production Y. Savings in this economy are assumed endogenously (equation 4.a) and are represented as a share of their respective income (S_p and S_w) which, in a closed economy setting, equates to investment (equation 6.a). Finally, equation 5.a assumes that the level of investment in this economy must match some sort of optimal investment equilibrium (I^*).

Overall, the model consists of six equations and five endogenous variables. As explained by Thissen (1998), this model can be mathematically solved by dropping equation five. Since Sen (1963), different model closures have emerged and nowadays they are generally classified into the following blocks:

- **savings-investment** identity.
- **current account balance** (open-economy setting).
- **government behaviour**.

Some authors, such as Gilbert and Tower (2013), define the previous blocks more compactly as macro-closures[5]. While, simultaneously, they distinguish other closures that are more focused on factor markets, micro-closures. For instance, whether prices of capital and stock of capital are assumed exogenous or endogenous; or especially the existence of unemployment in the model. Any of the previous closure blocks are ultimately linked to each other. For instance, the government's role in raising or lowering taxes affects disposable household income, which influences both

[5] Thissen (1998) provides additional model closures, but focused on macro CGE models.

consumption and investment. On the other hand, assuming a fixed level of current account deficit/surplus or allowing it to vary (endogenously) will also determine the total level of savings of the economy.

Following the above literature review on model closures, this section turns to an explanation and simulation of the main closures (savings-investment, government behaviour and current account balance) and micro-closures (unemployment). The shock in all models is the same and entails an increase in the capital endowment of 10%[6].

3.4.2 A brief literature review

This section reviews CGE models built to examine the welfare impact of policies or projects. We do not intend to provide a complete review of all possible CGE models with a welfare measure, but rather highlight selected models and examine their treatment of the closure. CGE models have been widely used in recent decades to model socially relevant questions. It has been argued that CGE models are not very useful unless the modeller pays attention to specific details, such as the level of sectoral and household disaggregation, assumptions made about the specification of key relationships, and the extent to which it represents a good approximation of the studied economy (De Maio, Stewart and Van Der Hoeven, 1999).

Thus, CGE models are often criticized for their reliance on the assumptions made in developing them. A key issue concerns the closure of the model, namely macro-economic, factor market, and foreign exchange account closures. Zalai and Révész (2016) rightly point out that despite the early warnings, the issue of model closure has been largely neglected in CGE studies. Taylor (2016) argues that while sectoral disaggregation is central to CGE analysis, the sectoral outcome of the model depends strongly on the closure of the model. Although it has long been established that model closure affects it qualitative outcomes (Taylor and Lysy, 1979; Rattsø, 1982; Adelman and Robinson, 1988; De Maio et al, 1999; Taylor, 2016), most models do not test the sensitivity of their results to model closure.

[6] All closures have been modeled in Mixed Complementarity Format (MCP) (Böhringer, et al, 2003). Under this format, the profit condition shows a complementarity condition with the activity variables, the market clearance condition with the price variables and the budget constraint with the income level.

De Maio et al, (1999) review CGE models developed to analyze the impact of adjustment policies on the poor in Africa and point out that macroeconomic and distributional outcomes of the models reflect assumptions made about the parameters, behavioural assumptions, and closure. The authors indicate that a CGE model is useful only if the assumptions reflect the realities of the economy concerned.

Dewatripont and Michel (1987) investigate the microeconomic foundations of the closure problem using a simple temporary competitive equilibrium model with a perfect foresight assumption. The authors demonstrate the implications of price expectations for the construction of a temporary equilibrium framework. Kilkenny and Robinson (1990) show that despite the relatively small role of agriculture in the U.S. economy, the nature of the impact of changes in agricultural policies depends, among others, on the degree of factor mobility and microeconomic closure assumptions.

Cloutier et al, (2008) provide a review of how the closure has been modelled in empirical CGE studies on the welfare implications of trade liberalization in developing countries. They argue that most studies have concluded that trade liberalization implied a positive effect on the overall welfare of an economy. However, equally, other studies found no aggregate welfare effect. Cloutier et al, (2008) pointed out that it is useful when evaluating findings to carefully examine the assumptions employed in the models concerning closure rules and market structure. The authors found that most models surveyed are closed in the (Neo) classical way, assuming fixed investment, endogenous wages, exogenous labour and full employment. Most importantly, despite its fundamental role in the construction and simulation process, some studies failed to provide sufficient guidance about how the model is closed.

Various authors have carried out comparative analyses of alternative macro and factor market closures, such as Taylor-Lysy (1979), Rattsø (1982), and De Melo and Robinson (1989). Adelman and Robinson (1988) construct a CGE model to estimate the distributional impact of macroeconomic adjustment programmes in developing countries. Their model incorporated different closures, namely neo-Keynesian, neoclassical, alternative macro closures for the balance of trade, and a variety of structuralist macro closure rules. The authors concluded that, the functional distribution (i.e. distribution between profit earners and wage earners), but not the size distribution of income, was sensitive to macro closure rules, and that the balance-of-trade closure was at least as important in determining distributional outcomes as the savings-investment closure.

Bourguignon, Branson and de Melo (1989) construct a CGE model that incorporates a financial sector with a treatment of asset markets that closely correspond to the stylized description of developing countries financial markets. They use the model to examine the effects of stabilization and structural adjustment mechanisms in emerging economies and conclude that the distribution of income and wealth is likely to be affected by alternative financial market closures. Rattsø (1982) claims that rather than building a general model and applying it to all sorts of policy-experiments, “the particular economic problems should inform both model-closure and model-formulation”. The importance of simulating CGE models is confirmed by Decaluwé and Monette (1988), who demonstrate that disturbances stemming from the supply or demand side of the economy may have different quantitative and qualitative impacts depending on the choice of a particular closure rule.

Most CGE models developed to examine welfare implication have focused on trade policies. However, there has been in recent years a growing volume of CGE models about the welfare impact of, for example, externalities and climate policy regimes (Juana, Strzepek and Kirsten, 2008; Twimukye and Matovu, 2009; Devarajan, Go, Robinson and Thierfelder, 2011; Pradhan and Ghosh, 2012; Dennis, 2016; Maddah, Berijanian and Ghazizadeh, 2018) or tourism expansion (Blake, Arbache, Sinclair and Teles, 2008; Wattanakuljarus and Coxhead, 2008; Li, Blake, and Thomas, 2013; Pratt, 2014; Njoya, Semeyutin, and Hubbard, 2020). A review of these studies reveals that the majority of them undertook a sensitivity test to explore the robustness of the model findings to key parameters and elasticities, concluding that the results in different sensitivity analyses do not differ significantly (in magnitude and direction) from those in the base case (Li, Blake and Cooper, 2011; Dennis, 2016). However, like in most CGE models, these studies did not incorporate an analysis to assess the sensitivity of the findings to different closure rules.

3.4.3 A formal analysis of the different *closures*

Investment-savings closure

Let's assume a closed economy, with one representative household, two factors (capital (K) and labour (L)) and two goods (X_1, X_2). Equations (1) and (2) denote the zero-profit condition equating total costs ($C_{Xi}(P_l, P_k)$) and total incomes P_{Xi} at their respective initial values ($\bar{X}_i$)[7]. Equation (3) represents the "zero-profit" condition of the representative household, where the total level of expenditure ($E(P_{x1}, P_{x2})$) equates welfare price index (P_w) with an initial value of $\bar{W}$. In equation 4, the investment decision is introduced in a similar way to the previous equations, with $I(P_{x1}, P_{x2})$ being the investment function and P_{inv} the price of the total investment; with an initial value of $\bar{I}$.

This economy faces a fixed level of capital ($\bar{K}$) and labour ($\bar{L}$), which are demanded as factors of production to produce X_1 and X_2 as shown in equations (5) and (6), where $\bar{X}_{i,k}$ and $\bar{X}_{i,l}$ denotes the initial demand of each sector (X_1 and X_2) concerning each factor (K, L). Both goods (X_1 and X_2) are finally consumed ($\bar{W}_{x1} \frac{\partial E(P_{x1},P_{x2})}{\partial P_{x1}} W$ and $\bar{W}_{x2} \frac{\partial E(P_{x1},P_{x2})}{\partial P_{x2}} W$) or invested ($\bar{I}_{x1} \frac{\partial I(P_{x1},P_{x2})}{\partial P_{x1}} I$ and $\bar{I}_{x2} \frac{\partial I(P_{x1},P_{x2})}{\partial P_{x2}} I$) according to equations (7) and (8), where $\bar{W}_{x1}, \bar{W}_{x2}, \bar{I}_{x1}$ and $\bar{I}_{x2}$ denotes the initial demand from household and investment concerning each good, respectively.

As shown in equation 10, the level of investment and consumption rely on household endowment, which is formed by the incomes obtained from labour ($w\bar{L}$) and capital ($r\bar{K}$) minus total savings ($\bar{S}$) available in this economy that is assumed fixed **(savings-driven closure**). As can be appreciated in equation (10), the existence of savings in this economy detracts final consumption from the representative households affecting the welfare (W) that can be attained (equation 9).

Finally, equation 11 equates investment (I) and savings ($\bar{S}$).

$$\bar{X}_1 C_{X1}(P_l, P_k) = P_{X1}\bar{X}_1 \quad (1)$$

$$\bar{X}_2 C_{X2}(P_l, P_k) = P_{X2}\bar{X}_2 \quad (2)$$

$$\bar{W} E(P_{x1}, P_{x2}) = P_w \bar{W} \quad (3)$$

[7] Variables with an upper bar denotes initial values. See table A.1 to see the initial values of all models.

$$\bar{I}I(P_{x1}, P_{x2}) = P_{inv}\bar{I} \tag{4}$$

$$\bar{K} = \sum_i \bar{X}_{i,k} \frac{\partial C_i(P_l,P_k)}{\partial P_k} X_i \tag{5}$$

$$\bar{L} = \sum_i \bar{X}_{i,l} \frac{\partial C_i(P_l,P_k)}{\partial P_l} X_i \tag{6}$$

$$\bar{X}_1 X_1 = \bar{W}_{x1} \frac{\partial E(P_{x1},P_{x2})}{\partial P_{x1}} W + \bar{I}_{x1} \frac{\partial I(P_{x1},P_{x2})}{\partial P_{x1}} I \tag{7}$$

$$\bar{X}_2 X_2 = \bar{W}_{x2} \frac{\partial E(P_{x1},P_{x2})}{\partial P_{x2}} W + \bar{I}_{x2} \frac{\partial I(P_{x1},P_{x2})}{\partial P_{x2}} I \tag{8}$$

$$\bar{W}W = \frac{M}{P_w} \tag{9}$$

$$M = r\bar{K} + w\bar{L} - \bar{S} \tag{10}$$

$$I = \bar{S} \tag{11}$$

The 11 endogenous variables are: $X_1, X_2, P_{x1}, P_{x2}, P_k, P_l, P_w, P_{inv}, W, I, M,$ for 11 equations. Thus, the model is "closed". Alternatively, total savings can be assumed endogenous by modifying the following equations in the model: First let's assume that total savings (S) vary according to the new equation (12) where $(1-\alpha)$ represents the share of total income (M) devoted to savings. As a result, equation (9) and (10) are rewritten as shown in equation (13) and (14), respectively. Now both investments and savings are endogenously determined within the model.

On the other hand, it should also be noted that if we now fix investment ($\bar{I}$) while keeping savings endogenous, then the model would also be closed (investment-driven closure).

$$S = (1-\alpha)\frac{M}{P_{inv}} \tag{12}$$

$$W = \alpha\frac{M}{P_w} \tag{13}$$

$$M = r\bar{K} + w\bar{L} \tag{14}$$

The differences in results when adopting one of these two savings rules can be better appreciated when simulating both models. Assuming Cobb-Douglas cost functions

($C_i(P_l, P_k) = P_l^{\gamma_i} P_k^{1-\gamma_i}$) for the production of both goods, investment ($I(P_{x1}, P_{x2}) = P_{x1}^{\beta} P_{x2}^{1-\beta}$) and household expenditure ($E(P_{x1}, P_{x2}) = P_{x1}^{\mu} P_{x2}^{1-\mu}$). The parameters of the models were calibrated according to the values shown in Table A.1 (see Appendix II).

As shown in Table 3, the economic impact varies in magnitude in both closures. For instance, the variation in sectoral production (X_1 and X_2) is 1.116 and 1.076 in both cases, respectively. Further, as expected, the capital-intensive sector (X_1) most benefits from the rise in capital endowment in both closures. And the price of capital reduces because of the rise in the supply of capital. Moreover, the variation in prices shows small differences in both cases. However, the largest differences emerge when analyzing the change in welfare and investment. Assuming a fixed level of savings allows for higher welfare gains (1.191), while assuming savings endogenously detract consumption attaining lower welfare gains (1.095), but increasing investment (1.095).

Table 3. Results of investment-savings closure (deviations from the initial equilibrium)

	Exogenous-savings	*Endogenous-savings*
X_1	1.116	1.116
X_2	1.076	1.076
W	1.191	1.095
I	1	1.095
P_{x1}	0.973	0.977
P_{x2}	1.009	1.013
P_k	0.904	0.908
P_l	1.085	1.090
P_h	0.991	0.995
P_{inv}	0.991	0.995

Government closure

The government fulfils the role of collecting taxes (both indirect and direct) while providing public goods and social transfers to households. Depending on which of these mechanisms are determined "outside" or "within" the model affects the economic adjustment and the results. These variables may also vary to achieve some level of surplus/deficit. Additionally, this closure may also interact and affect the investment-savings closure in two ways: firstly, indirectly by changing the endowment of the representative households, which, in the last term, will also affect the level of welfare. Secondly, directly, by allowing the government to invest. In any case, the government behaviour assumed will entail economic adjustments, which finally affect the outcome of the economy.

The government is introduced into the economy as follows:

$$\bar{G}G(P_{x1}, P_{x2}) = P_{x1}^{\alpha} P_{x2}^{1-\alpha} = P_G \bar{G} \tag{15}$$

$$GOV = i_{taxes} + transfers + surplus \tag{16}$$

$$\bar{X}_1 X_1 = \bar{W}_{x1} \frac{\partial E(P_{x1}, P_{x2})}{\partial P_{x1}} W + \bar{I}_{x1} \frac{\partial I(P_{x1}, P_{x2})}{\partial P_{x1}} I + \bar{G}_{x1} \frac{\partial G(P_{x1}, P_{x2})}{\partial P_{x1}} G \tag{17}$$

$$\bar{X}_2 X_2 = \bar{W}_{x2} \frac{\partial E(P_{x1}, P_{x2})}{\partial P_{x2}} W + \bar{I}_{x2} \frac{\partial I(P_{x1}, P_{x2})}{\partial P_{x2}} I + \bar{G}_{x2} \frac{\partial G(P_{x1}, P_{x2})}{\partial P_{x2}} G \tag{18}$$

$$M = r\bar{K} + w\bar{L} - \bar{S} + transfers \tag{19}$$

$$\bar{X}_1 C_{X1}(P_l, P_k) = P_l^{\gamma_i} P_k^{1-\gamma_i} = (P_{X1} + i_{taxes})\bar{X}_1 \tag{20}$$

$$\bar{X}_2 C_{X2}(P_l, P_k) = P_l^{\gamma_i} P_k^{1-\gamma_i} = (P_{X2} + i_{taxes})\bar{X}_2 \tag{21}$$

Equation (15) denotes the expenditure function of the government, where $\bar{G}$ and P_G represent the initial level of government expenditure and prices, respectively. The initial market clearance condition (7) and (8) must be redefined to accommodate the demand of goods from the government, equations (17) and (18). The government demands these goods, provides social transfers to households ($transfers$) and collects indirect taxes (i_{taxes}), according to equation (16). The production of goods X_1 and X_2, equations (1) and (2), needs to be modified to account for the indirect tax burden, equations (20) and (21). Finally, household endowment is also extended to include the social transfers

($transfers$) (equation, 21). Initially, indirect taxes are endogenous while transfers and surplus are fixed (**exogenous-transfers**).

Next, let's assume that the government decides to vary the social transfers ($transfers$) (**endogenous-transfers**) to keep the government surplus constant, as shown in equation 22. Alternatively, the surplus may also be assumed endogenously, yielding different results:

$$transfers = i_{taxes} - surplus \qquad (22)$$

Table 4. Results of investment-savings closure (deviations from the initial equilibrium)

	Exogenous-transfers	*Endogenous-transfers*
X_1	1.059	1.059
X_2	1.039	1.039
W	1.081	1.064
G	1.098	1
I	1	1
P_{x1}	0.991	0.991
P_{x2}	1.010	1.010
P_k	0.953	0.953
P_l	1.049	1.049
P_w	1	1
P_{inv}	1	1
$transfers$	1	1.098
$surplus$	1	1

As shown in Table 4, both closures, Exogenous-transfers and Endogenous-transfers, lead to equivalent economic adjustments in terms of production and prices, but they differ in the change in welfare, 1.081 and 1.064, respectively. As can also be appreciated, with fixed transfers, increases in tax collection also increases government consumption (1.098). However, by allowing transfers to vary, the level of consumption remains constant for the government.

Current-Account closure

The last macro closure refers to the consequence of adopting an open-economy framework. In this case, the main issue of concern relates to the existence of foreign deficit or surplus and the way of financing it. Moreover, it should be remembered that this deficit/surplus is directly linked to the level of savings in the economy, i.e., now in an open-economy situation, total savings is disentangled into domestic (S_d) and foreign savings (S_f) extending the investment-savings closure ($I = S_d + S_f$). At the same time, the government closure can also be affected when assuming public foreign deficit/surplus. The standard closure assumes a fixed current account surplus/deficit, while the exchange rate, imports and exports vary to match the initial surplus/deficit. This closure is widely used in small open economies where international prices are assumed exogenous and the availability of foreign savings is limited (Hosoe, et al, 2010; and Gilbert and Tower, 2013). Additionally, this closure also enhances the welfare analysis because it prevents from welfare changes caused by variations in the net foreign position (borrowing/lending from abroad). The open economy is modelled using equations (1) to (11) and adding the following equations (23-26) (exogenous current-account):

$$\bar{A}_i A_i = \bar{A}_i m_i^{\alpha_i} d_i^{1-\alpha_i} \; ; \text{ where } \; i = X_1, X_2 \tag{23}$$

$$\overline{EX}_{x1} P_{x1} = Pfx\overline{EX}_{x1} \tag{24}$$

$$\overline{EX}_{x2} P_{x2} = Pfx\overline{EX}_{x2} \tag{25}$$

$$M = r\bar{K} + w\bar{L} + \overline{deficit} \tag{26}$$

Equation (23) allows for imperfect substitution between imports (m_i) and domestic (d_i) goods/services (Armington, 1969) where α_i and $(1 - \alpha_i)$ represent the share of imports and domestic goods, respectively. Equations (24) and (25) denote the share of domestic production (X_1 and X_2) that is devoted to exports. Pfx denotes the real exchange rate, and $\overline{EX}_{x1}$and $\overline{EX}_{x2}$ the respective initial values of exports. Income constraint is also modified to encompass the inclusion of the current account deficit that is assumed fixed ($\overline{deficit}$) (equation, 26). The positive sign of the deficit denotes that the rest of the world is financing the economy. The adjustment of income constraint would be the same in the case of the current account surplus, but with a negative sign ($M = r\overline{K} + w\overline{L} - \overline{surplus}$), which implies that the economy is financing the rest of the world.

Combining equations (23-26) with equation (27) allows for endogenizing the current account deficit that was held constant in equation (26) (endogenous current-account). ∂_{x1} and ∂_{x2} denote the share of exports in the total production for X_1 and X_2, respectively. ini_{def}, ini_{x1}, ini_{x2}, ini_{m1}, ini_{m2} denote the initial values of the current account deficit, the initial domestic production of good X_1 and X_2, and the initial imports of X_1 and X_2, respectively.

$$f_{deficit}ini_{def} = (ini_{x1}\partial_{x1}X_1 + ini_{x2}\partial_{x2}X_2) - ini_{m1}\alpha_{x1}A_{x1} - ini_{m2}\alpha_{x2}A_{x2}$$
(27)

Finally, including equation (28) means that any change in savings is financed through foreign savings:

$$capflow\ Pfx = f\_deficit\ P_i \qquad (28)$$

According to Table 5, the largest changes are in prices, foreign deficit ($f\_deficit$) and real exchange rate. The presence of an endogenous current account implies that the foreign deficit rises due to the increase in the capital endowment. However, the change in welfare and investment remains the same when assuming financing investment with foreign savings. However, in the latter case, the economy is more expensive (P_w=1.028) than in the other two closures.

Table 5. Results of current-account closure (deviations from the initial equilibrium)

	Exogenous current-account	*Endogenous current-account*	*Endogenous current-account**
X_1	1.061	1.061	1.061
X_2	1.036	1.036	1.036
W	1.044	1.049	1.049
I	1.044	1.049	1.049
P_{x1}	0.979	0.979	1.012
P_{x2}	1.010	1.010	1.044
P_k	0.952	0.949	0.981
P_l	1.047	1.043	1.078
P_w	0.999	0.995	1.028
P_{inv}	0.999	0.995	1.028
$f\_deficit$	1	1.049	1.049
$capflow$	-	-	1.049
$real\_exchange\_rate$	1.004	0.995	1.028

*By financing investment through foreign savings

<u>Unemployment closure</u>

This closure is sometimes denoted as a micro closure. The existence of unemployment can easily be included in the CGE framework by extending income constraint as follows:

$$M = r\overline{K} + \frac{w\bar{L}}{1-U0} - \left(\frac{w\bar{L}}{1-U0}\right)Un \qquad (29)$$

$$P_l = P_w \qquad (30)$$

Equation (29) represents the income balance constraint, where Un is the variable that denotes unemployment level and $U0$ represents the initial unemployment level. Equation (30) denotes the price of labour (P_l) and assume that workers are willing to work when the variation of salaries equates the variation of the final price (P_w). Finally, the variable Un acts as a complementary variable of this equation[8]. It should be noted that the investment is omitted from this model for the sake of clarity. Thus, the model with an unemployment closure is based on equations (1) to (9), but omitting equation (4) and detracting the investment demand from equations (7) and (8). Now both goods are demanded as follows:

$$\bar{X}_1 X_1 = \bar{W}_{x1} \frac{\partial E(P_{x1},P_{x2})}{\partial P_{x1}} W \qquad (31)$$

$$\bar{X}_2 X_2 = \bar{W}_{x2} \frac{\partial E(P_{x1},P_{x2})}{\partial P_{x2}} W \qquad (32)$$

Table 6. Results of unemployment closure (deviations from the steady state)

	Full employment	*Unemployment*
X_1	1.059	1.100
X_2	1.039	1.100
W	1.049	1.100
P_{x1}	1.040	1
P_{x2}	1.060	1
P_k	1.001	1
P_l	1.101	1
P_w	1.050	1

As shown in Table 6, there are significant changes in production and welfare when assuming unemployment. The increase in capital endowment together with the

[8] An alternative common way of modeling unemployment is assuming a wage curve (Blanchflower & Oswald, 1995).

unemployment facilitate a multiplier effect. On the other hand, prices vary sharply with full employment.

3.5 Conclusion

The economic results of a CGE model will vary depending on the kind of closure assumed. These differences can be more marked when addressing real economies. However, there is no one-size-fits-all model closure since each relies on the kind of economic situation that best describes the particular simulation. For instance, assuming a fixed current account is widely-used in small open economies, where international prices are assumed as given, and the availability of foreign savings are limited.

Since the model closure conditions the results of the CGE model, then it may also condition any related result from the model, such as GDP or Equivalent Variation. The latter matters for the project appraisal, and if it varies with the model closure it may be a source of divergence with respect to Cost-Benefit Analysis.

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3.6 Appendix I

Application A code

Develop a CGE model in GAMS according to the following SAM:

SAM:

	Q1	Q2	Consumption	Income	**TOTAL**
PX1	50		50		100
PX2		50	50		100
PK	30	20		50	100
PL	20	30		50	100
TOTAL	100	100	100	100	

```
variables
U       total utility
;
U.l= 100 ;

positive variables

X1     consumption of the good 1
X2     consumption of the good 2
L1     labour demand production good 1
L2     labour demand production good 2
K1     capital demand production good 1
K2     capital demand production good 2
M      income of representative household
Q1     production of good 1
Q2     production of good 2
PX1    price of good 1
PX2    price of good 2
PK     price of capital
PL     price of labour ;

*initial values
X1.l=50;
X2.l=50;
Q1.l=50;
Q2.l=50;

parameter
*briefly, the shift parameter simply scale the utility to provide the same value as
the *consumption.
*it is not relevant in partial equilibrium, but is important in a general equilibrium
approach to *ensure the circular flow of income.
sigma       shift parameter of the utility function
gamma_q1    shift parameter of production function Q1
gamma_q2    shift parameter of production function Q2;



*the shift parameters are obtained inverting the respective function.

*shift parameter for utility
sigma   = U.l / (X1.l**0.5*X2.l**0.5)     ;
*shift parameter for production;
gamma_q1 = Q1.l/(30**0.6 * 20**0.4) ;
gamma_q2 = Q2.l/(20**0.4 * 30**0.6) ;
```

```
equations
utility        utility function
demand_X1      demand good 1
demand_X2      demand good 2
demand_L1      demand labour production good 1
demand_L2      demand labour production good 2
demand_K1      demand capital production good 1
demand_K2      demand capital production good 2
market_X1      market clearance for good X1
market_X2      market clearance for good X2
production_X1  production of good 1
production_X2  production of good 2
market_K       market clearance for capital K
market_L       market clearance for labour L
income_constraint  income constraint representative household
;

*according to the SAM, the share of good X1 and X2 in total consumption is,
respectively, 0.5 (50/100) and 0.5 (50/100)
utility..                 U    =e= sigma *  (X1**0.5*X2**0.5);
demand_X1..               X1   =e= 0.5 *M / PX1   ;
demand_X2..               X2   =e= 0.5 *M / PX2   ;


market_X1..               X1 =e= Q1;
market_X2..               X2 =e= Q2;

market_K..                50   =e= K1 + K2  ;
market_L..                50   =e= L1 + L2  ;

*the share of K and L in the production of good Q1 is 0.6 (30/50) and 0.4 (20/50),
respectively ;
production_X1..           Q1   =e= gamma_q1 * (K1**0.6*L1**0.4) ;
*the share of K and L in the production of good Q2 is 0.4 (20/50) and 0.6 (30/50),
respectively ;
production_X2..           Q2   =e= gamma_q2 * (K2**0.4*L2**0.6) ;

demand_L1..               L1   =e= (0.4*Q1*PX1)/PL   ;
demand_L2..               L2   =e= (0.6*Q2*PX2)/PL   ;
demand_K1..               K1   =e= (0.6*Q1*PX1)/PK   ;
demand_K2..               K2   =e= (0.4*Q2*PX2)/PK   ;


income_constraint..   M   =e=  PK*50   + PL*50  ;



model general_equilibrium  /all/;


*initial values
U.l= 100 ;
M.l= 100 ;
PX1.l= 1 ;
PX2.l= 1 ;
PK.l = 1 ;
PL.l = 1 ;
K1.l= 30 ;
L1.l= 20 ;
K2.l= 20 ;
L2.l= 30 ;
X1.l= 50 ;
X2.l= 50 ;
Q1.l= 50 ;
Q2.l= 50 ;

*replication of the initial equilibrium
option iterlim = 100 ;

solve general_equilibrium using NLP maximizing U;
```

Application B code

Develop a CGE model in MCP format, according to the previous SAM:

```
*defining variables
positive variables
X1      good X1
X2      good X2
PX1     price of good X1
PX2     price of good X2
PK      price of capital
PL      price of labour
PW      price of welfare (welfare index)
W       household
M       income household
;


equations
*zero profit
prf_X1         zero profit condition  X1
prf_X2         zero profit condition  X2
prf_W          zero profit condition  W
*market clearance
market_K       market clearance condition for capital
market_L       market clearance condition for labour
market_W       market clearance condition W
market_X1      market clearance condition X1
market_X2      market clearance condition X2
*income constraint
income_constraint   income  household;


prf_X1..         50*PX1  =E=  50 * PK**0.6*PL**0.4      ;
prf_X2..         50*PX2  =E=  50 * PK**0.4*PL**0.6      ;
prf_W..          100*PW  =E=  100* PX1**0.5*PX2**0.5    ;

market_K..            50  =E= 30*X1*PK**0.6*PL**0.4/PK  + 20*X2*PK**0.4*PL**0.6/PK   ;
market_L..            50  =E= 20*X1*PK**0.6*PL**0.4/PL  + 30*X2*PK**0.4*PL**0.6/PL   ;

market_W..           100*W  =E=  M/PW;
market_X1..          50*X1  =E=  50*W*PX1**0.5  *PX2**0.5/PX1;
market_X2..          50*X2  =E=  50*W*PX1**0.5  *PX2**0.5/PX2;


income_constraint..   M  =e=  PK*(50)   + PL*(50)  ;


model general_equilibrium /prf_X1.X1, prf_X2.X2, prf_W.W, market_X1.PX1,
market_X2.PX2, market_W.PW, market_K.PK ,market_L.PL, income_constraint.M  /;

*initial values
X1.l=1;
X2.l=1;
W.l=1;
PX1.l=1;
PX2.l=1;
PW.l=1;
PK.l=1;
PL.l=1;
M.l=100;


option iterlim = 0 ;

solve general_equilibrium using MCP;
```

Application C code

Develop a CGE model in MPSGE, according to the previous SAM:

```
$ONTEXT

$model:mpsge_intro

$sectors:

X1      !  activity level sector X1
X2      !  activity level sector X2
W       !  activity level sector W (hicksian welfare index)

$commodities:
PX1     ! price of good X1
PX2     ! price of good X2
PL      ! price of  labour
PK      ! price of capital
PW      ! price of welfare

$consumer:

M  ! income level representative household

*zero profit condition
$prod:X1 s:1

O:PX1 Q:50
I:PK Q:30
I:PL Q:20

$prod:X2 s:1

O:PX2 Q:50
I:PK Q:20
I:PL Q:30

$prod:W s:1

O:PW  Q:100
I:PX1 Q:50
I:PX2 Q:50


*The market clearance conditions are automatically generated by MPSGE when the model
is declared

*income constraint
$DEMAND:M

D:PW Q:100
E:PK Q:50
E:PL Q:50


$OFFTEXT

$SYSINCLUDE mpsgeset mpsge_intro

mpsge_intro.iterlim=0;

$INCLUDE  mpsge_intro.gen
SOLVE  mpsge_intro USING MCP;
```

3.7 Appendix II

Table A.1: Calibrated initial values of the CGE model*

	Investment-savings-closure	*Government-closure*	*Current-account closure*	*Unemployment closure*
γ_{x1}	0.6	0.6	0.6	0.6
γ_{x2}	0.4	0.4	0.4	0.4
β_{x1}	1	1	1	1
μ_{x1}	0.5	0.5	0.5	0.5
μ_{x2}	0.5	0.5	0.5	0.5
β	0.5	0.5	0.5	0.5
∂_{x1}	-	-	0.40	-
∂_{x2}	-	-	0.40	-
α_{x1}	-	-	0.45	-
α_{x2}	-	-	0.45	-
$\overline{EX}_{x1}, ini_{x1}$	-	-	40	-
$\overline{EX}_{x2}, ini_{x2}$	-	-	40	-
$\bar{m}_{x1}, ini_{m1}$	-	-	50	-
$\bar{m}_{x2}, ini_{m2}$	-	-	50	-
ini_{def}	-	-	20	-
$f\_deficit$	-	-	1	-
$capflow$	-	-	1	-
$\overline{X_1}$	100	100	100	100
$\overline{X_2}$	100	100	100	100
$\bar{I}$	100	100	100	-

*initially all prices are equal 1.

Table A.1 (continue): Calibrated initial values of the CGE model

	Investment-savings-closure	***Government-closure***	***Current-account closure***	***Unemployment closure***
$\bar{W}$	100	120	120	200
$\bar{M}$	100	120	120	100
$\bar{K}$	100	100	100	100
$\bar{L}$	100	100	100	100
$\bar{G}$	-	20	-	-
$\overline{GOV}$	-	20	-	-
$\overline{transfers}$	-	20	-	-
$\overline{\iota_{taxes}}$	-	20	-	-
∂_{x1}	-	-	40	-
∂_{x2}	-	-	40	-
α_{x1}	-	-	0.45	
α_{x2}	-	-	0.45	
A_{x1}	-	-	110	-
A_{x2}	-	-	110	-
m_{x1}	-	-	50	-
m_{x2}	-	-	50	-
d_{x1}	-	-	60	-
d_{x2}	-	-	60	-
$deficit$	-	-	20	-
$U0$	-	-	-	0.1
U	-	-	-	0.1