7 On the compatibility between CGE and CBA for project appraisal

Federico Inchausti-Sintes
Juan L. Eugenio-Martin
José M. Cazorla-Artiles

7.1 Introduction

This paper seeks a better understanding of the implications of employing Computable General Equilibrium (CGE) for project appraisal and its compatibility with Cost-Benefit Analysis (CBA). In this sense, it should be remembered that CGE and CBA models draw on the same economic theory, but employ different approaches. CGE models have predominantly focused on quantifying the economic effects of different policies in terms of Gross Domestic Product (GDP), inflation, sectoral changes, government surplus/deficit, unemployment, or current accounts (surplus/deficit); rather than welfare evaluation analysis, as is central in CBA.

This study provides a way to measure welfare impacts with CGE and discusses why and if so, how much, it may diverge with respect to CBA. The following cases are modelled: labour market with voluntary unemployment; labour market with involuntary unemployment, derived demand; derived demand with involuntary unemployment; derived demand with a negative externality; derived demand with non-competitive markets and a final model assuming different CGE *model closures*.

Impact assessment studies such as CGE need to be aware of the multiplier effects caused by any shock in the economy. Such effects need to be evaluated with respect to a counterfactual scenario, otherwise they may provide results with a positive bias in welfare terms. This paper discusses the role played by counterfactual scenarios for project appraisal with CGE. Finally, most of the cases are extended to an open-economy framework to check the consistency of the results and conclusions under this kind of scenario.

The theoretical approach is complemented with numerical examples for each model and extended to open-economy situations. Specifically, six examples are considered: labour market with voluntary unemployment; labour market with involuntary unemployment; derived demand; derived demand with involuntary unemployment; derived demand with a negative externality; and derived demand with non-competitive markets. As expected, the results confirm the theoretical supposition, highlighting that CGE can incorporate any of the opportunity costs stressed by CBA.

The remainder of the paper is organized as follows. Following this introduction, Section 7.2 starts demonstrating (theoretically) that CGE can handle certain market situations stressed in CBA. In Section 7.3, the four market situations (voluntary unemployment, involuntary unemployment, derived demand and externalities) are described; highlighting all aspects that the CGE model should capture from a welfare CBA perspective. In Section 7.4, a theoretical approach is developed to illustrate how show the way CGE can deal with welfare appraisal. Similarly, the main theoretical issues of concern are also addressed here to anticipate economic situations considered in a CGE framework to capture the welfare changes. In Section 7.5, the numerical examples are explained and simulated, together with the CGE counterfactual analysis. In Section 7.6, the relevance of the CGE counterfactuals is explained for welfare appraisal. Finally, the main findings and results are addressed in the divergence analysis section.

7.2 The foundations of the C-Bridge

This section starts "*bridging CBA and CGE"* (C-Bridge) by demonstrating theoretically that CGE can handle the market situations highlighted in CBA. This implies that, among other aspects, all economic changes that concur in the *primary* markets are implicitly included in the final demand of the representative/s household/s. The section demonstrates that the welfare change can be approached by income differences, with and without the project, by employing: *the Income Welfare Approach (IWA)*. Finally, the section highlights and demonstrates a series of theoretical issues of concern when conducting welfare analysis under CGE modelling. Furthermore, the theoretical consequences, in terms of welfare, of assuming multiple households and different *model closures* are explored.

7.2.1 Bridging CBA and CGE: a theoretical model

This section aims to show that the myriad of welfare variations that take place in different markets, as noted by CBA, are all captured in the final demand decision of the representative household when modelling the economy in CGE. Hence, the appraisal followed in CGE models can capture the welfare variations triggered by a project.

We start by eliciting one of the central assumptions in CGE models: the market clearance condition. This assumes that, in equilibrium, the quantities demanded equal the quantities supplied for all i markets, such as:

$$Demand_i = Supply_i \qquad (1)$$

7.2.1.1 Closed economy without government

Let's assume a closed economy without government, one representative household, and two factors of production (K and L), without intermediate demands. The supply-side of the market clearance condition (equation 1) can now be represented more succinctly as $Supply_i = Y_i = F_i(K_i, L_i)$, where the supply/production of good (i) depends positively on the factors of production (K_i and L_i), which are combined according to the technology (F_i), to produce good i. Similarly, the demand-side depends positively on income level (M) and negatively on prices (P_i).

$$D_i(M, P_i) = Y_i = F_i(K_i, L_i) \qquad (2)$$

Assuming that the production function $F_i(K_i, L_i)$ is a homogenous function, then $F_i(K_i, L_i)$ can be decomposed into demand for the factors of production as follows: $tF_i(K_i(r, P_i), L_i(w, P_i)) = \frac{\partial F_i(K_i(r,P_i),L_i(w,P_i))}{\partial K_i} K_i + \frac{\partial F_i(K_i(r,P_i),L_i(w,P_i))}{\partial L_i} L_i$,

where $\frac{\partial F_i(K_i,L_i)}{\partial L_i} = \frac{W}{P_i} = w$ and $\frac{\partial F_i(K_i,L_i)}{\partial K_i} = \frac{R}{P_i} = r$, where W and R denote wages and the cost of capital, respectively, while w and r denote the respective real values, and t the degree of homogeneity. Let's assume for simplicity that $t = 1$. Thus, equation $Y_i = F_i(K_i, L_i)$ can be written as: $Y_i = rK_i + wL_i$ and equation (2) stands now as follows:

$$D_i(M, P_i) = Y_i = F_i(K_i, L_i) = rK_i + wL_i \qquad (2.1)$$

By multiplying both sides of equation (2.1) by the respective market prices and adding over i goods/services, it yields:

$$\sum_{i=1}^{n} D_i(M,P_i)\,P_i = \sum_{i=1}^{n} Y_i\,P_i \quad (3)$$

where $\sum_{i=1}^{n} D_i(M,P_i)\,P_i$ represents the total expenditure (E) and $\sum_{i=1}^{n} Y_i\,P_i = (r\sum_{i=1}^{n} K_i + w\sum_{i=1}^{n} L_i)P_i$ the total income constraint in nominal terms (M). Hence, when the circular flow of income holds, the change that takes place in one market is finally captured in the income constraint of the representative household[1].

Let's take the case of the development of an economic project. We distinguish between two situations: 0 and f, which represent the initial equilibrium without the project (0) and the final equilibrium when the project has been implemented (f). Moreover, let the income level M vary between both equilibria. Now, equation (2.1) can be disentangled as follows:

$$D_i^0(M^0,P_i^0) = r^0K_i^0 + w^0L_i^0 \quad (3.1)$$

$$D_i^f\left(M^f,P_i^f\right) = r^fK_i^f + w^fL_i^f \quad (3.2)$$

Adding equations (3.1) and (3.2) by goods yield. the respective Walrasian equilibrium (Varian, 1992) is:

$$\sum_{i=1}^{n} D_i^0(M^0,P_i^0) = \sum_{i=1}^{n} \frac{\partial F_i^0(K_i(r,P_i),L_i(w,P_i))}{\partial K_i} K_i^0 + \sum_{i=1}^{n} \frac{\partial F_i^0(K_i(r,P_i),L_i(w,P_i))}{\partial L_i} L_i^0 \quad (3.3)$$

$$\sum_{i=1}^{n} D_i^f\left(M^f,P_i^f\right) = \sum_{i=1}^{n} \frac{\partial F_i^f(K_i(r,P_i),L_i(w,P_i))}{\partial K_i} K_i^f + \sum_{i=1}^{n} \frac{\partial F_i^f(K_i(r,P_i),L_i(w,P_i))}{\partial L_i} L_i^f$$

(3.4)

Multiplying the right-hand side of equations (3.3) and (3.4) by the respective market prices yields expenditure levels, such as: $E_0 = \sum_{i=1}^{n} P_i^0 D_i^0(M^0,P_i^0)$ and $E_f = \sum_{i=1}^{n} P_i^f D_i^f(M^f,P_i^f)$.

Similarly, by multiplying the left-hand side of these equations by their respective prices of factors (wage and price of capital) generate income levels (income constraints), such as: $M_0 = \sum_{i=1}^{n} \frac{\partial F_i^0(K_i(r,P_i),L_i(w,P_i))}{\partial K_i} K_i^0 + \sum_{i=1}^{n} \frac{\partial F_i^0(K_i(r,P_i),L_i(w,P_i))}{\partial L_i} L_i^0$,

[1] The demonstration can easily be relaxed to include more than one representative household with identical and homogenous tastes. In this case, the welfare variation is obtained by adding the respective equivalent variations. Instead of assuming identical and homogenous tastes, Deaton and Muellbauer (1980) opt for a weaker assumption by considering that all consumers have income-expansion paths that are linear and parallel (Engle´s curves).

and $M_f = \sum_{i=1}^{n} \frac{\partial F_i^f(K_i(r,P_i),L_i(w,P_i))}{\partial K_i} K_i^f + \sum_{i=1}^{n} \frac{\partial F_i^f(K_i(r,P_i),L_i(w,P_i))}{\partial L_i} L_i^f$.

Finally, subtracting equation (3.3) from (3.4) we obtain that:

$$E_0 - E_f = M_0 - M_f \quad (3.5)$$

In essence, equation (3.5) shows that successive changes that may take place in one market (the primary market) affect the whole economy. And thus, they are included in the representative agent's expenditure functions.

However, the project's magnitude of the project is not the only factor that affects income level in the economy. For instance, the public sector not only demands goods and services, but also collects taxes and transfers social subsidies to households, which affects the income of the economy. Similarly, in an open-economy setting, the economy not only generates an inflow (imports) and outflow (exports) of goods and services with the rest of the world; but also affects the disposal income of the economy by paying and selling such imports and exports, respectively: which, in turn, affects the current account deficit or surplus. Hence, social welfare must be extended to include the aforementioned income effects in the economy.

7.2.1.2 Open economy with government

Now the total demand of the economy is also composed of export demand (X) and government demand (D^G), and household demand (D^H). Similarly, the supply-side is extended because of imports[2] (m). In sum, and differentiating again between the initial and final equilibrium yields the following demand and supply expressions:

$$D_i^0(M^0, P_i^0) = D_i^{H,0}(M^{H,0}, P_i^0) + D_i^{G,0}(M^{G,0}, P_i^0) + X_i^0$$

$$Y_i^0 = F_i^0(K_i^0, L_i^0, m_i^0) = r^0 K_i^0 + w^0 L_i^0 + m_i^0$$

$$D_i^f(M^f, P_i^f) = D_i^{H,f}(M^{H,f}, P_i^f) + D_i^{G,f}(M^{G,f}, P_i^f) + X_i^f$$

$$Y_i^f = F_i^f(K_i^f, L_i^f, m_i^f) = r^f K_i^f + w^f L_i^f + m_i^f$$

Since $D_i^0(M^0, P_i^0) = Y_i^0$ and $D_i^f(M^f, P_i^f) = Y_i^f$, then:

$$D_i^{H,0}(M^{H,0}, P_i^0) = r^0 K_i^0 + w^0 L_i^0 + m_i^0 - X_i^0 - D_i^{G,0}(M^{G,0}, P_i^0) \quad (3.6)$$

[2] It should be noted that upper-case denotes income; whereas lower-case refers to imports.

$$D_i^{H,f}\left(M^{H,f},P_i^f\right) = r^f K_i^f + w^f L_i^f + m_i^f - X_i^f - D_i^{G,f}\left(M^{G,f},P_i^f\right) \quad (3.7)$$

Adding equations (3.6) and (3.7) by goods we obtain that:

$$\sum_{i=1}^{n} D_i^{H,0}(M^{H,0},P_i^0) = \sum_{i=1}^{n} \frac{\partial F_i^0(K_i(r,P_i),L_i(w,P_i),IM_i(Pm_i,P_i))}{\partial K_i} K_i^0 + \sum_{i=1}^{n} \frac{\partial F_i^0(K_i(r,P_i),L_i(w,P_i),IM_i(Pm_i,P_i))}{\partial L_i} L_i^0 + \sum_{i=1}^{n} \frac{\partial F_i^0(K_i(r,P_i),L_i(w,P_i),IM_i(Pm_i,P_i))}{\partial IM_i} m_i^0 - \sum_{i=1}^{n} \frac{\partial F_{EX_i}^0(Y_i)}{\partial Y_i} Y_i^0 - \sum_{i=1}^{n} D_i^{G,0}(M^{G,0},P_i^0) \quad (4.1)$$

with $F_{EX_i}(Y_i)$ denoting exports production.

$$\sum_{i=1}^{n} D_i^{H,f}\left(M^{H,f},P_i^f\right) = \sum_{i=1}^{n} \frac{\partial F_i^f(K_i(r,P_i),L_i(w,P_i),IM_i(Pm_i,P_i))}{\partial K_i} K_i^f + \sum_{i=1}^{n} \frac{\partial F_i^f(K_i(r,P_i),L_i(w,P_i),IM_i(Pm_i,P_i))}{\partial L_i} L_i^f + \sum_{i=1}^{n} \frac{\partial F_i^f(K_i(r,P_i),L_i(w,P_i),IM_i(Pm_i,P_i))}{\partial IM_i} m_i^f - \sum_{i=1}^{n} \frac{\partial F_{EX_i}^f(Y_i)}{\partial Y_i} Y_i^f - \sum_{I=1}^{n} D_i^{G,f}\left(M^{G,f},P_i^f\right) \quad (4.2)$$

Multiplying both sides of equations (4.1) and (4.2) by their respective market prices, we obtain the respective expenditure and income functions (income constraints), such that:

$$E_0^H = RF_0 + ca_0 - G_0 = M_0 \quad (4.3)$$

$$E_f^H = RF_f + ca_f - G_f = M_1 \quad (4.4)$$

where M_0 is now composed by the rent of labour and capital ($RF_0 = \sum_{i=1}^{n} \frac{\partial F_i^0(K_i,L_i,IM_i)}{\partial K_i} K_i^0 + \sum_{i=1}^{n} \frac{\partial F_i^0(K_i,L_i,IM_i)}{\partial L_i} L_i^0$), the current account position ($ca_0 = \sum_{i=1}^{n} \frac{\partial F_i^0(K_i,L_i,M_i)}{\partial M_i} M_i^0 - \sum_{i=1}^{n} \frac{\partial F_{EX_i}^0(Y_i)}{\partial Y_i} Y_i^0$) and finally, the total public spending ($G_0 = \sum_{i=1}^{n} D_i^{G,0}(M^{G,0},P_i^0)$). Similarly, RF_f, ca_f and G_f denotes the counterpart of the previous expressions, but in relation to the final equilibrium (f).

Subtracting (4.3) from (4.4) yields:

$$E_f^H - E_0^H = (RF_f + ca_f - G_0) - (RF_0 + ca_0 - G_f) \quad (4.6)$$

Equation (4.6) is equivalent to equation (3.5) but assumes an open economy with the government. Hence, as noted, both equations are based on showing that the welfare change enhanced by a project can be calculated by focusing directly on final demand, omitting the successive changes that occurred in the economy's other markets.

7.2.2 Approaching the equivalent variation by income variation: the income welfare approach (IWA)

An additional result in terms of welfare is implicit when analyzing equations (3.5) and (4.6): *"the welfare variation, measured by the equivalent variation, can also be calculated by analyzing the changes that take place in the income constraint"*. The formal demonstration is addressed below.

Proposition: *The Equivalent Variation (EV) can be approached by the difference between the income level before and after the project's implementation, in a general equilibrium framework.*

Proof:

Let's begin by eliciting the mathematical expression of the equivalent variation:

$$EV = e(P^0, U^f) - e(P^0, U^0)$$

where superscript 0 and f denote the situation with and without the project, respectively, and e represents an expenditure function that depends on prices (P) and utility level (U). Through the circular flow of income, total expenditure equals total income, such that: $e(P^0, U^0) = M^0$ and $EV = e(P^0, U^f) - M^0$.

By taking into account that the expenditure function is separable into prices and utility, then $e(P^0, U^f)$ can be rewritten as $e(P^0)U^f$, such that:

$$EV = e(P^0)U^f - M^0$$

Both $e(P^0)$, which denotes the consumer price index, and M^0 are known, whereas utility level U^f remains unobservable[3]. Fortunately, its values can be retrieved from a standard maximizing utility problem, where the problem's first-order condition can be written as:

[3] This is also unobservable; however, it is embedded within M^0.

$$\frac{\partial U}{\partial X_i} = \frac{P_{X_i}}{P_U}$$

Adding the first-order condition by goods, it yields:

$$\sum_{i=1}^{n} \frac{\partial U}{\partial X_i} = \sum_{i=1}^{n} \frac{P_{X_i}}{P_U}$$

Since utility is a homogenous function, according to the Euler theorem, we can obtain that: $\sum_{i=1}^{n} \frac{\partial U}{\partial X_i} = \frac{tU}{\sum_{i=1}^{n} X_i}$, where t denotes the degree of homogeneity[4]. Hence, substituting the previous expression $\sum_{i=1}^{n} \frac{\partial U}{\partial X_i} = \sum_{i=1}^{n} \frac{P_{X_i}}{P_U}$ and rearranging it, yields $U = \frac{\sum_{i=1}^{n} X_i P_i}{t P_U}$, which means that the utility level equals expenditure level ($\sum_{i=1}^{n} X_i P_i$) in real terms ($M^f = \frac{\sum_{i=1}^{n} X_i P_i}{t P_U}$). Replacing U^f with the latter expression in the equivalent variation, yields that:

$EV = e(P_0)M^f - M^0$.

Assuming that the initial prices equal 1[5] in the equilibrium, then this implies that $e(P_0) = 1$.

Hence, $EV = M^f - M^0$ [6].

This result coincides with Johansson (2022) when deriving general equilibrium cost-benefit rules for large projects through expenditure functions. Likewise, it is also comparable to the approach used by Johansson and Kriström (2016) when employing the indirect utility function to conduct welfare evaluations in CBA.

Further, it is worth highlighting that the result holds under any economic or market situation (involuntary unemployment, non-competitive markets[7], existence of a

---

[4] In general, the production function is assumed to be homogenous of degree 1 ($t = 1$).

[5] It should be noted that assuming initial prices different to 1 implies a monotonic transformation of M^f and M^0, but the EV does not vary.

[6] The demonstration can be easily relaxed to include more than one representative household. In this case, the welfare variation is obtained by calculating the total equivalent variation of all households considered. Likewise, when assuming an economy with a public sector, its equivalent variation should also be considered in the total equivalent variation (see Section 7.6).

[7] In essence, involuntary unemployment is already reflecting a non-competitive market situation.

government or an open economy framework, among others), except when assuming negative externalities, as shown below.

7.3 The implications of distortions for the C-Bridge

The presence of distortions in the economy is the main source of potential divergences between CGE and CBA. In this paper we study the distortions caused by voluntary unemployment, involuntary unemployment, derived demand and externalities. We formally extend the analysis by highlighting, and anticipating, the main theoretical issues of concern when dealing with some of these situations in a CGE framework. This Section explains the underpinnings of such distortions.

7.3.1 Derived demand

CBA provides a convenient shortcut when the project under analysis causes a reduction in the cost of an input employed by other sectors in the economy. If the markets operate in a competitive environment without distortions, the analysis should concentrate on the input market to calculate the social welfare; thereby avoiding double-counting (de Rus, 2021). Alternatively, the welfare analysis may focus on output instead of input markets, yielding the same welfare result. However, in real case situations, it is easier to collect information in the input market instead of collecting information in all markets affected by the cost reduction.

The use of the primary market where the first effect of the project occurs is not restricted to the case of an input derived demand. Observed demand in one market concentrates valuable economic information of multiple effects in other markets. For instance, in the case of substitution effects in other markets (secondary markets) with price changes, Boardman, Greenberg, Vining and Weimer (2018) show how observed demand in the primary market correctly captures the substitution effect between the primary market and other markets.

From a CGE perspective, derived demand does not seem to provide any shortcut, since this approach requires the whole economy to be modelled. Besides, regarding welfare, CGE models focus on the output markets (representative household). Thus, once again the question is whether, in the case of derived demand, a CGE model correctly evaluates the welfare impact of a project by concentrating on final demand, or if it eventually results in double-counting by not drawing a distinction between economic impacts and welfare changes.

Derived demand is demand for a factor of production that occurs as a result of the demand for another intermediate or final good. If the project under analysis causes, for instance, a reduction in the cost of an input employed by other sectors in the economy, then the welfare impact can be measured either by focusing on the input market/s or, alternatively, by focusing on the output markets that demand this intermediate good. Otherwise, if both kinds of markets are added (input and output markets), then the welfare evaluation would result in double-counting. This result holds when assuming competitive markets without distortions (taxes or subsidies, for example) in the economy. We can more formally prove this result when addressing a CGE modelization.

Proposition: *In the case of derived demand, welfare variation in the output market equals the welfare variation that occurs in the input market.*

Proof:

Let's assume an economy with Y_i goods/sectors, two factors of production (K and L) supplied inelastically to the market and where the production of one good (Y_Z) is entirely demanded as an intermediate good by other sectors. Thus, the production functions of the economy are: $Y_i = F_i(K, L)$ when $i = Z$, and $Y_i = F_i(K, L, Z)$, otherwise. Finally, let's assume a variation in the production of good Y_Z such as $\Delta Y_Z > 0$, with $\Delta P_Z < 0$. As noted, in order to avoid double-counting, CBA holds that, either, we should focus on the output markets (Y_Y and Y_X), or, the input markets (Y_Z) to compute the welfare variation.

Let's approach the welfare variation by employing the variation of the total surplus (ΔS): $\Delta S = \sum_{i=1}^{n} \int_{P_0}^{P_f} D_i^i(M, P_i)$ where D_i denotes the final demand of good i, which depends positively on income (M) and negatively on its own price (P_i)[8].

[8] Total surplus includes both consumer and producer surplus. It should be noted that in CGE a single representative household/agent is usually assumed.

By the market clearance condition, demand equals supply, so that:

$D_i^i(M, P_i) = Y_i$

And the production Y_i depends on the factors of production and the intermediate good Y_Z:

$Y_i = F_i(K_i, L_i, Z_i)$

According to the Euler equation, the production equation can be written as follows:

$$Y_i = \frac{\partial Y_i}{\partial L_i} L_i + \frac{\partial Y_i}{\partial K_i} K_i + \frac{\partial Y_i}{\partial X_i} Z_i$$

By including this expression in terms of the variations in production (ΔY):

$\Delta Y = \sum_{i=1}^{n} \int_{w_0}^{w_f} D_L^i(I_i, w) + \sum_{i=1}^{n} \int_{r_0}^{r_f} D_K^i(I_i, r) + \sum_{i=1}^{n} \int_{P_0^f}^{P_Z^f} D_Z^i(I_i, P_Z)$ where $D_L^i(I_i, w)$ and $D_K^i(I_i, r)$ denotes labour and capital demand from the economic sectors (output markets), which depend positively on the income of each sector (I), and negatively on the prices of both factors, w and r, respectively, and $D_Z^i(I_i, P_Z)$ represents the demand of good *Z*.

Keeping in mind that $D_i^i(M, P_i) = Y_i$, then, we get that $\Delta S = \sum \Delta S_i = \sum \Delta Y_i$; where ΔS denotes the total surplus variation, ΔS_i represents the total surplus variation by good i and ΔY_i the production variation by good i.

$$\Delta S = \sum_{i=1}^{n} \int_{P_0}^{P_f} D_i^i(M, P_i) = \sum_{i=1}^{n} \int_{w_0}^{w_f} D_L^i(I_i, w) + \sum_{i=1}^{n} \int_{r_0}^{r_f} D_K^i(I_i, r) + \sum_{i=1}^{n} \int_{P_0^f}^{P_Z^f} D_Z^i(I_i, P_Z)$$

Use of the total surplus variation relies on assuming the income level as constant, which implies that $\sum_{i=1}^{n} \int_{w_0}^{w_f} D_L^i(I_i, w)$ and $\sum_{i=1}^{n} \int_{r_0}^{r_f} D_K^i(I_i, r)$ equal zero. Hence, the variation in total surplus collapses to:

$$\Delta S = \sum_{i=1}^{n} \int_{P_0}^{P_f} D_i^i(M, P_i) = \sum_{i=1}^{n} \int_{P_0^f}^{P_Z^f} D_Z^i(I_i, P_Z)$$

where $\sum_{i=1}^{n}\int_{P_0^f}^{P_Z^f} D_Z^i(I_i, P_Z)$ equals the variation in production of sector Y_Z (ΔY_Z) (input market) such as:

$$\Delta Y_Z = \sum_{i=1}^{n} \int_{P_0^f}^{P_Z^f} D_Z^i(I_i, P_Z)$$

Hence:

$\Delta S = \Delta Y_Z$ (ΔS^{output}=ΔS^{input})

Proving that the welfare change can be calculated either by focusing on the output markets (ΔS^{output}) or on the input markets (ΔS^{input}), as stated by CBA. However, this result relies on assuming the income level is constant. When the latter does not hold, its effect in terms of welfare is captured by using the equivalent variation, which is the standard welfare measure in CGE (Hosoe, Gasawa and Hashimoto, 2010).

7.3.1.1 Equivalent variation: conditions for convergence in CBA and CGE

The case of derived demand allows us to highlight the potential welfare equivalence when conducting CBA and CGE. As noted, this measure provides a convenient shortcut when a project triggers no income effect. However, as soon as the project generates the latter, then the Equivalent Variation and Surplus Variations differ[9].

7.3.1.2 Input market multiplicative effect (*Pme*)

Another important result emerges when delving into the case of intermediate demand and analyzing welfare change; not in terms of total surplus variation or production variation, but in terms of the multiplicative effect in welfare.

Specifically, we want to test the following proposition:

Proposition: *In case of intermediate demand, the multiplicative effect observed in the input market coincides with the multiplicative welfare effect triggered in the economy.*

Proof:

[9] Willig (1976) shows under what conditions the magnitude of the error between both welfare measures is not significant.

Let's firstly introduce the new notation, reflecting that the variables are now measured in multiplicative terms, such as: $\tilde{X} = \frac{X_f}{X_0}$ where X_0 and X_f represent initial and final values, respectively.

Considering that, $EV = \Delta M$, in multiplicative terms, we obtain:

$$\widetilde{EV} = \widetilde{M}$$

Similarly, the expression $EV = e(P^0, U^f) - e(P^0, U^0)$ can also be transformed so that:

$$\widetilde{EV} = \tilde{e}$$

Since total expenditure equals total demand, in multiplicative terms, we obtain that:

$$\tilde{e} = \sum_{i=1}^{n} \widetilde{Y_\iota}\widetilde{P_\iota}$$

At the same time, according to the Euler theorem, total production can be decomposed into its factors of production, such that:

$$\frac{\partial Y_i}{\partial L_i}\widetilde{L_\iota} + \frac{\partial Y_i}{\partial K_i}\widetilde{K_\iota} + \frac{\partial Y_i}{\partial X_i}\widetilde{X_\iota}$$

Hence:

$$\sum_{i=1}^{n} \widetilde{Y_\iota}\widetilde{P_\iota} = \sum_{i=1}^{n} \left(\frac{\partial Y_i}{\partial L_i}\widetilde{L_\iota} + \frac{\partial Y_i}{\partial K_i}\widetilde{K_\iota} + \frac{\partial Y_i}{\partial X_i}\widetilde{X_\iota}\right)\widehat{P_\iota}$$

Assuming that, either, there is no variation in the demand of labour and capital in output markets ($\widetilde{L_\iota} = \widetilde{K_\iota} = 0$), or, that total variation is zero ($\sum_{i=1}^{n} \frac{\partial Y_i}{\partial L_i}\widetilde{L_\iota} + \frac{\partial Y_i}{\partial K_i}\widetilde{K_\iota} = 0$) then the equivalent variation collapses to:

$$\widetilde{EV} = \sum_{i=1}^{n} \widetilde{Y_\iota}\widetilde{P_\iota} = \sum_{i=1}^{n} \left(\frac{\partial Y_i}{\partial X_i}\widetilde{X_\iota}\right)\widetilde{P_\iota}$$

Since the production of intermediate good equals its respective demand from output markets, then: $\widetilde{Y_Z} = \sum_{i=1}^{n} \widetilde{X_\iota}$. So that, $\sum_{i=1}^{n} \widetilde{X_\iota} = \sum_{i=1}^{n} \widetilde{Y_\iota}$. This implies that: $\widetilde{Y_Z} = \sum_{i=1}^{n} \widetilde{Y_\iota}$. And finally, that: $\widetilde{EV} = \widetilde{Y_Z}\,\widetilde{P_Z}$[10].

This demonstration considerably simplifies estimation of a project's welfare effect by simply calculating the *Pme* triggered in the market under analysis. However, albeit

[10] See footnote 6.

different to the previous case when neglecting the income effect in the demonstration of variation in intermediate demand, this new result also relies on a key assumption. Specifically, it holds that either there are no variations in the sectoral demand of factors in the output markets ($\widetilde{L_\iota} = \widetilde{K_\iota} = 0$), or that the variation exists, but its total net effect is zero ($\sum_{i=1}^{n}(\widetilde{L_\iota} + \widetilde{K_\iota}) = 0$). However, what does this assumption imply? and, how realistic is it?

The first case, $\widetilde{L_\iota} = \widetilde{K_\iota} = 0$, would be implicit when assuming, for instance, full employment of both labour and capital, and perfect labour mobility. In this situation, $\widetilde{L_\iota} = \widetilde{K_\iota} = 0$ holds, meaning that the economic impact of the project in output markets can be omitted from the welfare evaluation and simply focus on the input one. Thus, $\widetilde{EV} = \widetilde{Y_Z}\,\widetilde{P_Z}$holds.

The second case, $\sum_{i=1}^{n}(\widetilde{L_\iota} + \widetilde{K_\iota}) = 0$, can be better contextualised when assuming unemployment. On the one hand, it should be noted that now, $\widetilde{L_\iota} = \widetilde{K_\iota} = 0$, does not hold, because the output markets are also capable of increasing their production by seeking unemployed workers. On the other, the constraint: $\sum_{i=1}^{n}(\widetilde{L_\iota} + \widetilde{K_\iota}) = 0$, has to be reformulated as follows: $\sum_{i=1}^{n}(\widetilde{L_\iota} + \widetilde{K_\iota}) + \widetilde{U} = 0$, where $\widetilde{U}$ denotes the multiplicative change in unemployment that takes place in output markets. As can be appreciated, it is reasonable to assume that the employment created by output markets equates to a reduction in unemployment, so that $\sum_{i=1}^{n}(\widetilde{L_\iota} + \widetilde{K_\iota}) + \widetilde{U} = 0$. As a result, $\widetilde{EV} = \widetilde{Y_Z}\,\widetilde{P_Z}$ holds again. A similar conclusion is obtained when introducing a government, or assuming non-competitive market conditions.

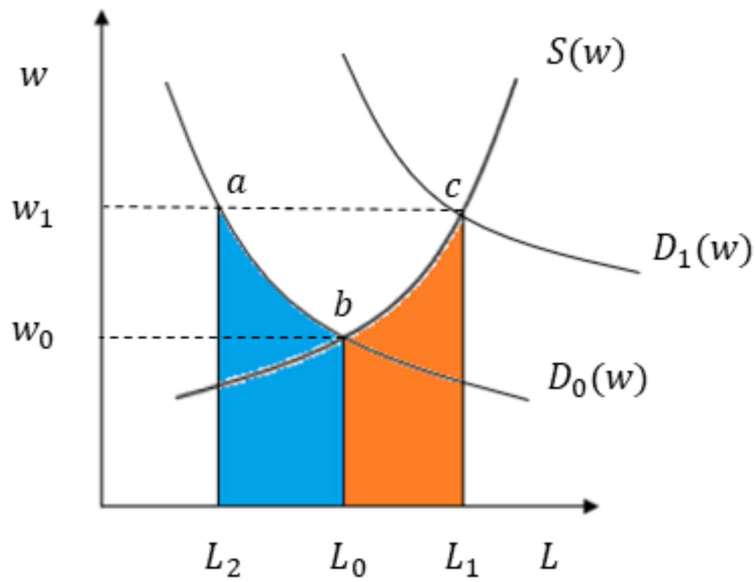
However, $\widetilde{EV} = \widetilde{Y_Z}\,\widetilde{P_Z}$ no longer holds when considering a deficit or surplus in the current account (open economy). In this situation, the economy can expand (deficit) or contract (surplus) resources beyond the constraints of a closed economy. For instance, let's assume a small open economy with a deficit, where output markets demand imports as commodities. Now, $\sum_{i=1}^{n}(\widetilde{L_\iota} + \widetilde{K_\iota}) = 0$ is $\sum_{i=1}^{n}\widetilde{L_\iota} + \sum_{i=1}^{n}\widetilde{K_\iota} + \sum_{i=1}^{n}\widetilde{m_\iota} + \widetilde{def} = 0$, where $\widetilde{m_\iota}$ denotes the multiplicative effect on import demand in the output markets and $\widetilde{def}$ represents the multiplicative effect on the deficit of the economy. Now, if these markets increase their demand for imports ($\widetilde{m_\iota}$) as a result of the effect of the project in the input market, $\widetilde{def}$ rises as well, causing $\sum_{i=1}^{n}\widetilde{L_\iota} + \sum_{i=1}^{n}\widetilde{K_\iota} + \sum_{i=1}^{n}\widetilde{m_\iota} + \widetilde{def} \neq 0$. Hence, $\widetilde{EV} \neq \widetilde{Y_Z}\,\widetilde{P_Z}$. Summarizing, $\widetilde{EV} = \widetilde{Y_Z}\,\widetilde{P_Z}$ works adequately in any economic situation, except when considering a deficit or surplus in the current account. In this case, $\widetilde{EV} \neq \widetilde{Y_Z}\,\widetilde{P_Z}$.

7.3.2 The labour market: voluntary and involuntary unemployment

In the economic evaluation of projects, resources are valued at the social opportunity cost (de Rus, 2021). In general, as soon as there is competitive market behaviour, the price reflects the social opportunity cost of the resource. However, in cases where the market price does not reflect the opportunity cost, CBA uses shadow prices.

Figure 1 illustrates a competitive labour market. In this example, the project increases the demand for labour from D_0 to D_1, which increases the demand of workers by $L_1 - L_0$ units. The welfare implications for the $L_1 - L_0$ new workers should be evaluated by considering the opportunity cost of their leisure (area bcL_1L_0). On the other hand, the $L_0 - L_2$ workers hired by the project come from other sectors and their opportunity cost should take into account such loss (area: abL_0L_2). The same reasoning is applied when assuming the existence of a factor supplied perfectly inelastically to the market, such as land.

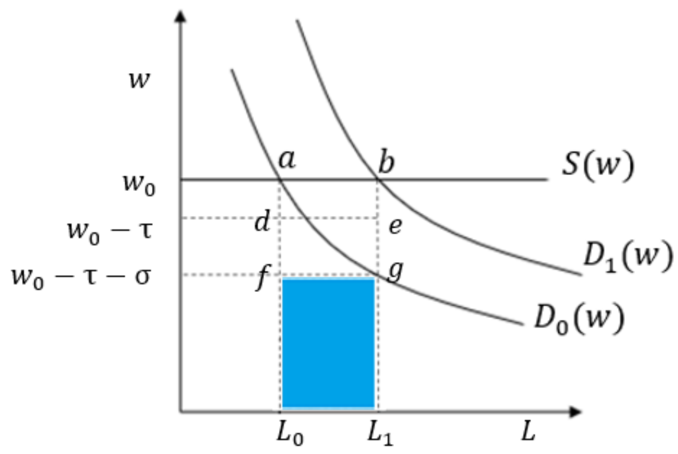
Figure 1. Labour markets and shadow price



Finally, the social opportunity cost may also differ when assuming involuntary unemployment (Figure 2). For instance, without income taxes (τ) or unemployment benefits (σ), the clearing salary (w_0) captures the social opportunity cost whose total change is denoted by the area abL_1L_0. However, by introducing taxes and/or unemployment benefits in this market, the salary w_0 no longer represents the social opportunity cost, but $w_0 - \tau$ and/or $w_0 - \tau - \sigma$, where τ denotes the income tax and σ unemployment benefits. In both cases, the social opportunity cost is represented by the areas deL_1L_0 and fgL_1L_0, respectively.

CGE models are capable of modelling any of these labour market situations. However, they do not calculate the welfare changes that are taking place under these different labour market situations as CBA does, but focus directly on the estimation of the representative household's welfare change. In sum, the question is whether the opportunity costs elicited by CBA, in any of these labour market situations, are implicitly incorporated into the welfare of the household in a CGE framework, or whether the former requires additional adjustments to address them correctly.

Figure 2. Involuntary unemployment and shadow price



7.3.2.1 <u>Open economy and involuntary unemployment with derived demand</u>

The existence of imports in the output markets or involuntary unemployment in the economy may also cause that $\Delta S \neq \Delta Y_Z$. The underlying idea is that the latter equality does not only rely on assuming that income level as constant, but also that the total sum of the variation in the demand of factors equals zero: $\sum_{i=1}^{n}(\Delta L_i + \Delta K_i) = 0$

Let's extend the technology of the output sectors (Y_Y and Y_X) to include imports (m_i) as follows:

$Y_i = F_i(K_i, L_i, Z_i, m_i)$

In consequence, total variation in production stands as:

$$\Delta Y = \sum_{i=1}^{n} \int_{w_0}^{w_f} D_L^i(I_i, w) + \sum_{i=1}^{n} \int_{r_0}^{r_f} D_K^i(I_i, r) + \sum_{i=1}^{n} \int_{r_0}^{r_f} D_m^i(I_i, Pm) + \sum_{i=1}^{n} \int_{P_0^f}^{P_Z^f} D_Z^i(I_i, P_Z)$$

where Pm denotes import prices. However, even assuming the income level as constant if, for instance, $\Delta m_i \neq 0$, then it implies that $\sum_{i=1}^{n}(\Delta L_i + \Delta K_i + \Delta m_i) \neq 0$, causing that:

$\Delta Y = \sum_{i=1}^{n} \int_{r_0}^{r_f} D_m^i(I_i, Pm) + \sum_{i=1}^{n} \int_{P_0^f}^{P_Z^f} D_Z^i(I_i, P_Z)$. Hence, $\Delta S \neq \Delta Y_Z$.

The existence of involuntary unemployment, or simply that $\sum_{i=1}^{n}(\Delta L_i + \Delta K_i) \neq 0$ [11], would also cause ($\Delta S^{output} \neq \Delta S^{input}$).

7.3.3 Derived demand with a negative externality

7.3.3.1 A negative externality

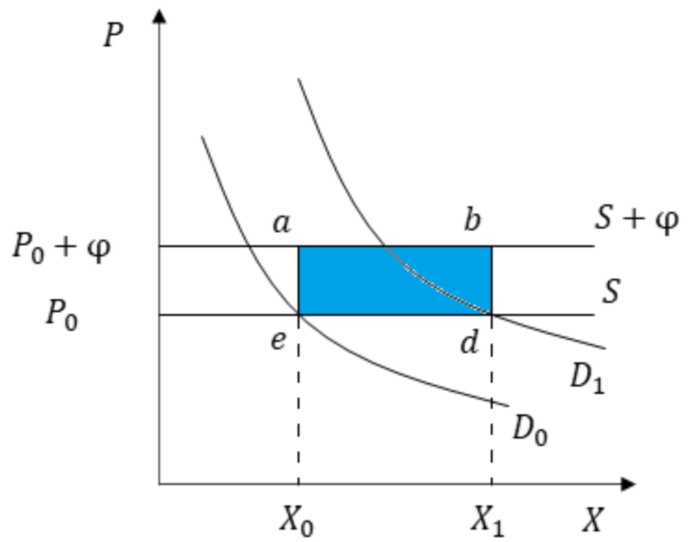
This example extends the previous by assuming the existence of a negative externality in the output market. As shown in Figure 3, the equilibrium in the market is e where the price paid is P_0 and the quantity exchanged is X_0. However, the "true" price (including the externality) is $(P_0 + \varphi)$, where φ denotes the externality. However, the market does not clear at this price, but at P_0; resulting in a greater provision of the good than is socially desirable. In other words, the externality operates as a subsidy in this good's production, i.e. the true marginal cost is $P_0 + \varphi$, but this good is produced assuming a marginal cost equal to P_0. When demand shifts upward from D_0 to D_1 because of the effect of the project in the input market, then the output market clears at

[11] This situation arises, for instance, when assuming an increase in capital productivity, instead of total factor productivity in the input market.

d, but the market is incurring a social cost represented by the area ($abde$) that has to be subtracted from the social welfare.

Once again, the question is whether the opportunity cost elicited by CBA in this case, is implicitly incorporated in the welfare of a household in a CGE framework, or whether the former requires additional adjustments, in order to address it correctly.

Figure 3. Negative externalities



As said, an important issue of concern when conducting project appraisal is the existence of externalities in the economy. In terms of welfare analysis, a difficulty arises when noting that its presence implies that one good is supplied above what is socially desirable because it does not internalize the social cost caused during its production, which means that the private cost is lower than the social cost. As a result, the presence of an externality not only causes an increase in production but, more importantly, it also affects the economy's welfare. Next, we more formally examine this consequence.

Let's assume that a closed economy produces according to the following production function $Y = Y^{\beta}F(K, L)$, where Y denotes total production, K and L represent capital and labour, respectively, $F(K, L)$ operates under constant returns to scale and β is a parameter that reflects the degree of the externality (Liu and Turnovsky, 2005; Romer, 1986; and Lucas, 1988). Hence, Y is homogenous of degree: $\frac{1}{1-\beta} > 1$.

According to the Euler theorem, a production function with constant returns to scale can be written in variations, as follows:

$$\Delta Y = \frac{\partial F(K,L)}{\partial K}\Delta K + \frac{\partial F(K,L)}{\partial K}\Delta L$$

Considering that $Y = Y^{\beta}F(K,L)$, then the previous equation can be rewritten as follows:

$$\Delta Y = \Delta Y^{\beta}\frac{\partial F(K,L)}{\partial K}\Delta K + \Delta Y^{\beta}\frac{\partial F(K,L)}{\partial L}\Delta L$$

Assuming that both factors of production are paid according to their marginal productivity, it implies that $\frac{\partial F(K,L)}{\partial K} = r$ and $\frac{\partial F(K,L)}{\partial L} = w$. Hence:

$$\Delta Y = \Delta Y^{\beta} r\Delta K + \Delta Y^{\beta} w\Delta L$$

Knowing that, by the circular flow of income, $\Delta M = r\Delta K + w\Delta L$:

$$\Delta Y = \Delta Y^{\beta}\Delta M$$

Operating this expression yields that:

$$\Delta Y = \Delta M^{\frac{1}{1-\beta}}.$$

Furthermore, recalling from the previous section that $EV = \Delta M$, we obtain that:

$$\Delta Y = EV^{\frac{1}{1-\beta}}.$$

This demonstrates that the existence of externalities in the production of a good causes a distortion with respect to welfare. It should be noted that when $\beta = 0$ (no externality), total variation in production equals the Equivalent Variation ($\Delta Y = \Delta M = EV$).

7.3.3.2 Externalities and intermediate demand

Now the analysis briefly returns to testing the consistency of the welfare measures previously demonstrated, but now assuming externalities. As already proven, in the case of derived demand, the total surplus variation equals the variation in production in the input market: $\Delta S = \Delta Y_Z$.

Let's extend the technology of the output sectors Y_Y to include the externality as follows:

$Y_i = Y_i^{\beta} F_i(K_i, L_i, Z_i)$ when $i = Y$

As a result, the factors' demand functions take the form:

$\frac{\partial Y_{i=Y}}{\partial K_{i=Y}} Y_{i=Y}^{\beta} = \frac{r}{P}$; $\frac{\partial Y_{i=Y}}{\partial L_{i=Y}} Y_{i=Y}^{\beta} = \frac{w}{P}$; $\frac{\partial Y_{i=Y}}{\partial Z_{i=Y}} Y_{i=Y}^{\beta} = \frac{P_Z}{P}$

Thus, total variation in production is:

$$\Delta Y = \Delta Y_{i=Y}^{\beta} \left(\sum_{i=1}^{n} \int_{w_0}^{w_f} D_L^i(I_i, w) + \sum_{i=1}^{n} \int_{r_0}^{r_f} D_K^i(I_i, r) + \sum_{i=1}^{n} \int_{P_0^f}^{P_Z^f} D_Z^i(I_i, P_Z) \right)$$

Remembering that $D_i^i(M, P_i) = Y_i$ yields, as previously, that $\Delta S = \Delta Y$. However now, even assuming a constant income level, $\Delta S > \Delta Y_Z$ ($\Delta S^{output} > \Delta S^{input}$), because ΔY is now affected by the externality of sector Y ($\Delta Y_{i=Y}^{\beta}$). This result means that approaching the welfare change using ΔS^{output} yields a biased result in the presence of externalities in the output markets, whereas ΔY_Z (ΔS^{input}) is not affected by the latter, and thus reports a reliable welfare value. This is solved in CBA by taking into account the presence of distortions in the good markets or ignoring them when the value of the distortions are common to the counterfactual.

Regarding the IWA and Pme, both are also affected by the externality by simply recalling that their calculus rely on the change of income level, and that, as previously shown, the externality causes the following effect on the latter: $\Delta Y = EV^{\frac{1}{1-\beta}} = M^{\frac{1}{1-\beta}}$. Thus, the externality must be subtracted from both measures to provide an unbiased welfare result.

7.3.4 Other theoretical issues of concern

There are two additional issues of concern when conducting welfare analysis: multiple households and *model closure,* also known as *macroclosure*. Both aspects have already been covered in Inchausti-Sintes and Njoya (2022) so this section briefly highlights their main consequences in terms of welfare. In any case, the EV continues to provide a correct welfare approach in any of these economic situations.

7.3.4.1 Multiple households

In general, CGE considers a single representative household. However, depending on the kind of project under analysis, more households may be required. As highlighted by Varian (1992), when the household functional form fulfils the Gorman norm, exact lineal aggregation is granted. This means that aggregate welfare remains constant, no matter the kind of income distribution.

However, when different types of households have got different income elasticities (different marginal social utility of income), the Gorman norm no longer holds, meaning that the aggregate welfare varies with changes in the income endowment (see, Varian, 1992 or Deaton and Muellbauer, 1980).

7.3.4.2 Model closure

This issue only affects CGE models because it refers to the *closure* of the foreign position, the governmental position and the investment-savings rule (Hosoe, et al, 2010; and Gilbert and Tower, 2013). The choice of the *closure* of any of these economic situations affects welfare. As stated by Inchausti-Sintes and Njoya (2022), a model is mathematically "closed" when we have sufficient independent equations to explain the endogenous variables (Gilbert and Tower, 2013). Further, the choice of exogenous and endogenous variables also determines the computability and complexity of the model (Hosoe, et al, 2010).

The first is generally imposed by the economy under analysis (i.e., practically all CGE models are built upon the small open economy assumption, meaning that the foreign position (zero deficit, deficit or surplus) is fixed). In other words, the capacity of the economy to attract foreign savings is limited. Thus, this is not an issue that can be freely determined by the modeller.

The second refers to how the government determines its deficit, surplus or zero deficit. Broadly speaking, the government collect taxes, make transfers to households, and spends on consumption, which, together, determine the budgetary position. Thus, depending on which of these items are exogenously or endogenously determined, the welfare will vary in consequence.

Finally, the investment-savings rule refers to the way of modelling the investment decision of the economy: assuming exogenous or endogenous investment. Paraphrasing Gilbert and Tower (2013), when the former holds (investment-driven decision), the welfare variation can be interpreted as the effect for a given level of investment in future consumption. However, according to the same authors, if the analysis seeks to determine how the project under analysis impacts the economy through its effect on savings, then the latter may be chosen (savings-driven decision).

In the following section, the theoretical welfare appraisal is applied and tested in a CGE framework for different markets situations introduced in the cases studies: a labour market with voluntary unemployment, a labour market with involuntary unemployment, derived demand and derived demand with a negative externality. Additionally, the case of derived demand is complemented assuming involuntary unemployment and non-competitive markets. Moreover, the case studies are extended to open economy situations to check the consistency of the results and conclusions. Finally, the empirical implication of *model closure* is covered in section 7.6, which also illustrates the capacity of CGE to conduct counterfactual scenarios.

7.4 CGE models

As shown in Inchausti-Sintes and Njoya (2022), any CGE model relies on fulfilling three conditions: zero benefit, market clearance conditions and income balance (Böhringer, Rutherford and Wiegard, 2003). Zero benefit means that the value per activity must be equal to or greater than the value of its output. Market clearance implies that the supply of any good/service must be equal to or exceed the demand for these goods/services. At the same time, the demand can be disentangled into intermediate and final demand. Finally, the income balance of each institution (government or households, mainly) must be equal to or exceed their final demands. The CGE models developed in this paper are built upon these three conditions, as explained below.

7.4.1 Voluntary unemployment

Let's assume a closed economy without government, with two sectors each producing one single output (Y_i, with $i = X\ and\ Y$), (Y_i). The two sectors employ capital and labour as factors of production. More specifically, the former is assumed sector-specific ($\widehat{K}_i$) and the latter is perfectly mobile among sectors (L_i). Thus, the economic decision adopted by each sector can be summarized according to the following maximising problem:

Sectoral behaviour

$$\text{(1.M1)} \quad max_{Y_i,\widehat{K}_i,L_i}\,(P_iY_i) - \left(r_i\widehat{K}_i + wL_i\right)$$

$$subject\ to: Y_i = f(\widehat{K}_i, L_i)$$

where P_i denotes the price of sector/good i, r_i denotes the cost of capital in sector/good i and w is the wage. The solution to this problem yields the demand of capital ($\widehat{K}_j$) and labour L_j by sectors. Assuming a Cobb-Douglas production function with constant returns to scale: $Y_i = f\left(X_{ij}, \widehat{K}_i, L_i\right) = \widehat{K}_i^{\alpha_i^{\widehat{K}}} L_i^{\alpha_i^{L}}$ the factors demand function takes the following functional forms: $\widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i} P_iY_i$; and $L_i = \frac{\alpha_i^{L}}{w} P_iY_i$.

Finally, thanks to the production duality problem (Mas-Colell, Whinston and Green, 1995) (1.M1) is equivalent to the cost minimizing problem. Substituting the conditional demands for the factors of production obtained from this problem in the objective function yields the cost function ($C_i(r_i, w, Y_i) = r_i^{\alpha_i^{\widehat{K}}} w^{\alpha_i^{L}} Y_i$). This function allows us to obtain an expression of the cost of production associated with the level of output (Y_i). Finally, this function form, together with the income and the zero-profit condition provides that: $P_iY_i - r_i^{\alpha_i^{\widehat{K}}} w^{\alpha_i^{L}} Y_i$=0.

Voluntary unemployment

In a standard CGE model, labour is supplied perfectly inelastically to the market (vertical labour supply); ensuring that it is employed by the economic sectors (full employment). However, the existence of involuntary employment requires us to introduce an adjustment into the model. The labour endowment is owned by the households (representative household), but they have now to decide between leisure and labour, implying that the labour supply is upward sloping. As a result, the consumption of leisure must be introduced in the household's consumption basket. Mathematically, both economic behaviours are accommodated in a CGE model, as follows:

Labour supply

The labour supply adopts the following form and transforms leisure into labour supply, such that:

$$\text{(2.M1)}\ \ max_{L_s,L_H}\ (wL_s) - (P_L L_H)$$

$$subject\ to: L_s = f(L_H)$$

According to problem 2.M1, the labour endowment (L_H) is supplied to the market as L_s (labour supply), w denotes the wage, and P_L represents the opportunity cost of labour (the cost of leisure). When w rises, it causes an increase in the opportunity cost of leisure (P_L), meaning that more workers are willing to exchange leisure for working hours. If we assume a Cobb-Douglas production function with constant returns to scale, then the optimal demand of labour is: $L_H = \frac{1}{P_L} wL_s$. The cost function associated with this problem is $P_L L_s$ and the zero-profit condition is $wL_s - P_L L_s = 0$.

Household behaviour

All the production obtained from the maximization problem (1.M1) is devoted to satisfying each household's demand, which is constrained by their disposal income (M). The bundle of goods demanded from households are now composed by the two goods produced by the two sectors (C_X and C_Y) and the "consumption" of *leisure.* The household consumption decision is represented as follows:

$$\text{(3.M1)}\ max_{C_X,C_Y,leisure}\ U(C_X, C_Y, leisure)$$

$$subject\ to: M = \sum_{i=A}^{B} P_i C_i + P_L leisure$$

where U denotes total utility which comprises the consumption of both goods ($C_X\ and\ C_Y$) and the enjoyment of *leisure* (C_L). Finally, P_i and P_L denote the prices of each good and the opportunity cost of leisure (cost of labour), respectively. Assuming a Cobb-Douglas utility function with constant returns to scale ($U = C_X^{\beta_X} C_Y^{\beta_Y} C_L^{\beta_L}$), the demand functions of problem 3.M1 are:

$C_X = \frac{\beta_X}{P_X} M$; $C_Y = \frac{\beta_Y}{P_Y} M$; and ; $C_L = \frac{\beta_L}{L} M$

The expenditure function associated with this problem is: $P_U = P_X^{\beta_X} P_Y^{\beta_Y} P_L^{\beta_L}\, U$. This function represents the consumer price index of the economy, and it is usually employed as *numeraire* in CGE modelling to deflate all other prices (Wing, 2004). The zero-profit condition is: $P_U U - P_X^{\beta_X} P_Y^{\beta_Y} P_L^{\beta_L} U = 0$.

General equilibrium

The zero-profit conditions: $P_i Y_i - r_i^{\alpha_i^{\hat{K}}} w^{\alpha_i^L} Y_i$=0, $wL_s - P_L L_s = 0$ and $P_U U - P_X^{\beta_X} P_Y^{\beta_Y} P_L^{\beta_L} U = 0$. And the demand: $\hat{K}_i = \frac{\alpha_i^{\hat{K}}}{r_i} P_i Y_i$, $L_i = \frac{\alpha_i^L}{w} P_i Y_i$, $L_H = \frac{1}{P_L} w L_s$, $C_X = \frac{\beta_X}{P_X} M$, $C_Y = \frac{\beta_Y}{P_Y} M$ and $C_L = \frac{\beta_L}{L} M$ have to be complemented by additional equations (market clearance conditions for goods and factors, and income constraint) to obtain a full characterization of this economy.

(4.M1) $Y_i = C_i$

(5.M1) $\bar{L} = L_H + C_L$

(6.M1) $L_H = L_S$

(7.M1) $\bar{K_\imath} = \hat{K}_i$

(8.M1) $\bar{K_\imath} = \sum_{i=1}^{n} \hat{K}_i$

(9.M1) $M = \sum_{j=A}^{B} r_j \hat{K}_i + w\bar{L}$= $\sum_{i=A}^{B} P_i C_i + P_L C_L$

where Equation (4.M1) ensures that the production of each good i (Y_i) is demanded as final goods (C_i). Equations (5.M1), (6.M1) and (7.M1) ensure that the sectors entirely demand the labour and capital owned by the households. Equation (8.M1) assumes that all the sector-specific capital equals total capital endowment. Finally, equation (9.M1) represents the income balance constraint of the representative household. Table 1 summarizes the equations employed in this model.

Table 1. The CGE model equations, with voluntary unemployment.

Zero profit
$P_iY_i - r_i^{\alpha_i^{\widehat{K}}} w^{\alpha_i^L} Y_i$=0
$(wL_s) - (P_LL_s)$=0
$P_UU - P_X^{\beta_X} P_Y^{\beta_Y} P_L^{\beta_L} U = 0$
Market clearance condition
$\overline{K}_i = \widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i} P_iY_i$
$L_s = \sum_{i=1}^{3} \frac{\alpha_i^L}{w} P_iY_i$
$\overline{L} = \frac{1}{P_L} wL_s + \frac{\beta_L}{L} M$
$X = C_X \frac{\beta_X}{P_X} M$
$Y = C_Y = \frac{\beta_Y}{P_Y} M$
$Y_X + Y_Y = C_X + C_Y$
Income constraint
$M = \sum_{i=1}^{n} r_i\widehat{K}_i + w\overline{L} = \sum_{i=A}^{B} P_iC_i + P_LC_L$; being $\overline{K_\imath} = \sum_{i=1}^{n} \widehat{K}_i$

7.4.2 Open economy[12] with voluntary unemployment

Let's now distinguish between the tradable and non-tradable sectors to distinguish between exportable and non-exportable sectors/goods, respectively. In this sense, sector X will be regarded as tradable, meaning that its production is now disentangled into

[12] The open-economy assumption relies on considering a small open economy whose foreign position (zero deficit, deficit or surplus) is assumed to be fixed (standard *closure* in a small open economy).

domestic and exportable production. Whereas sector Y, the non-tradable sector, continues to operate domestically. Moreover, both sectors now demand imports as inputs (m_i). Finally, the economy represented in the model is considered small, implying that the international export (Pe^w) and import prices (Pm^w) are given exogenously and take the value 1. Thus, the domestic export and import prices are: $Pe = er\ Pe^w$ and $m = er\ Pm^w$; where er refers to the exchange rate. The export sector can be described by the following profit maximizing problem:

$$(10.M1)\quad max_{E_X, Y_X}\,(Pe_X E_X) - (P_X Y_X)$$

$$subject\ to: E_X = F(Y_X)$$

where E_X denotes exports of good Y_X. The solution to this problem yields the intermediate demand of the export sector to reach export production E_X. Once again, assuming a Cobb-Douglas production function with constant returns to scale for $E_X = f(Y_X)$, and solving problem (10.M1), yields the demand: $Y_X = \frac{1}{P_X} Pe_X E_X$. Using the cost function to form the zero-profit condition yields: $(Pe_X E_X) - (P_X E_X)$=0

Similarly, the sectoral behaviour must be rewritten to include the demand of imports as intermediate inputs:

$$(11.M1)\quad max_{Y_i}\,(P_i Y_i) - \left(r_i \widehat{K}_i + wL_i + P_m m_i\right)$$

$$subject\ to: Y_i = f(\widehat{K}_i, L_i, m_i)$$

Assuming again, a Cobb-Douglas production function: $Y_i = f\left(\widehat{K}_i, L_i, M_i\right)\widehat{K}_i^{\alpha_i^K} L_i^{\alpha_i^L} M_i^{\alpha_i^M}$, the demand functions of $\widehat{K}_i$, L_i and m_i from each sector i takes the following form: $\widehat{K}_i = \frac{\alpha_i^K}{r_i} P_i Y_i$; $L_i = \frac{\alpha_i^L}{w} P_i Y_i$; and $M_i = \frac{\alpha_i^M}{Pm} P_i Y_i$. The zero-profit condition associated with this problem is: $P_i Y_i$- $r_i^{\alpha_i^K} w^{\alpha_i^L} {P_m}^{\alpha_i^M} Y_i = 0$.

Finally, the bundle of goods demanded by the representative household are the same as in the closed economy. However, the income constraint (equation 9.M1) has to be rewritten to accommodate the current account position: $M = \sum_{j=A}^{B} r_j \widehat{K}_j + w\bar{L} + er\ ca$, where ca denotes the current account (exports minus imports) and the magnitude of which can be zero, positive or negative; implying zero deficit, deficit, or surplus with the rest of the world, respectively. Similarly, equation (4.M1) also has to accommodate the existence of exports and imports in the economy: $Y_Y = C_Y - m_Y$ and $Y_X = C_X + E_X - m_X$. Finally, equations (5.M1), (6.M1) (7.M1) and (8.M1) continue to hold in the small open economy framework. Table 2 shows the CGE model equations, assuming a small open economy with voluntary unemployment.

Table 2. The equations of the small open economy CGE model with voluntary unemployment.

Zero profit
$(Pe_Y E_Y) - (P_Y E_Y)$=0
$P_i Y_i$- $r_i^{\alpha_i^K} w^{\alpha_i^L} {P_m}^{\alpha_i^M} Y_i = 0$
$(wL_s) - (P_L L_s)$=0
$P_U U - P_X^{\beta_X} P_Y^{\beta_Y} P_L^{\beta_L} U = 0$
Market clearance condition
$\overline{K}_i = \widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i} P_i Y_i$
$L_s = \sum_{i=1}^{3} \frac{\alpha_i^L}{w} P_i Y_i$
$\bar{L} = \frac{1}{P_L} w L_s + \frac{\beta_L}{L} M$
$Y_Y = \frac{1}{P_Y} Pe_Y E_Y$
$m_i = \frac{\alpha_i^M}{P_m} P_i Y_i$
$C_X = \frac{\beta_X}{P_X} M = X$
$C_Y = \frac{\beta_Y}{P_Y} M = Y$
$Y_Y = C_Y - M_Y$
$Y_X = C_X + E_Y - M_X$
$Pe = er\ Pe^w$
$Pm = er\ {P_m}^w$
$E_Y - m_Y - m_X = ca$
Income constraint
$M = \sum_{j=A}^{B} r_j \widehat{K}_j + w\bar{L} + er\ fp = \sum_{i=A}^{B} P_i C_i + P_L C_L$; being $\overline{K_\iota} = \sum_{i=1}^{n} \widehat{K}_i$

7.4.3 Involuntary unemployment and unemployment benefits

The previous model must be slightly adjusted to cope with involuntary unemployment (classical unemployment). The latter is introduced in the model by assuming a lower bound on real wages (the legal minimum wage), where the minimum wage equals the consumer price index ($w^{min} \geq P_U$). Furthermore, in this new labour market situation, the representative household cannot decide between leisure and work. Hence, equation 2.M2 is not applicable. Similarly, $\bar{L} = L_w$, whereas the household decision is rewritten as follows:

Household behaviour

$$(1.M2)\ max_{C_X,C_Y}\ U(C_X, C_Y)$$

$$subject\ to: M = \sum_{i=A}^{B} P_i C_i$$

Household income constraint, denoted by equation (9.M1), is also accommodated as: $M = \sum_{j=A}^{B} r_j \widehat{K}_j + P_{Ln}\left(\frac{\bar{L}}{(1-\overline{Un})}\right) - P_{Ln}\left(\frac{\bar{L}}{(1-\overline{Un})}\right) Un + sub\ \overline{Transfers}$ (9.M2), where P_{Ln} refers to the salary net of taxes ($P_{Ln} = w - \tau$), $\overline{Un}$ and $\overline{Transfers}$ are parameters denoting the initial unemployment rate and the initial level of unemployment benefits, respectively; whereas Un and sub are variables referring to the unemployment rate and unemployment benefit rates, respectively. The latter is positively related to the former ($sub = Un$). The representative household continues to operate with a Cobb-Douglas utility function ($U = C_X^{\beta_X} C_Y^{\beta_Y}$). Hence the demand functions of C_X and C_Y are: $C_X = \frac{\beta_X}{P_X} M$; $C_Y = \frac{\beta_Y}{P_Y} M$. Finally, the zero-profit condition is: $P_U U - P_X^{\beta_X} P_Y^{\beta_Y} U = 0$.

The production-side of this economy continues to produce with the same technology and under the same market conditions. However, labour demand in both sectors is levied with an income tax (τ) collected by the government which, at the same time, demand final goods and transfer subsidies to the households. Hence, the zero-profit condition is: $P_i Y_i$- $r_i^{\alpha_i^K} (w + \tau)^{\alpha_i^L} {P_m}^{\alpha_i^M} Y_i$ and the demand of factors are: $\widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i} P_i Y_i$ and $L_i = \frac{\alpha_i^L}{w+\tau} P_i Y_i$.

As a result, the wage observed by the representative household (P_L) is net of taxes, whereas equation (4.M1) also includes the demand for government goods (G_i), such that: $Y_i = C_i + G_i$.

Government behaviour

Government behaviour can be written as follows:

$$(2.M2) \quad max_{G_X,G_Y}\, U^G(G_X, G_Y)$$

$$subject\ to\colon M^G = \sum_{i=A}^{B} P_i G_i$$

Where $M^G = taxes - sub\ \overline{Transfers} = \sum_{i=A}^{B} P_i G_i$

where G_X and G_Y refer to the government consumption of goods Y_X and Y_Y, and $taxes$ refers to the income taxes collected ($\tau w(L_X + L_Y)$). The utility function of the public sector takes a Cobb-Douglas function form: $U^G = G_X^{\beta_X^G} G_Y^{\beta_Y^G}$. Hence, the zero-profit condition is: $P_{U^G}U^G - P_X^{\beta_X^G} P_Y^{\beta_Y^G} P_L^{\beta_L} U^G = 0$. Table 3 shows all the equations of the CGE model with involuntary unemployment.

Table 3. The equations of the CGE model with involuntary unemployment and unemployment benefits.

Zero profit
$P_i Y_i$- $r_i^{\alpha_i^K}(w+\tau)^{\alpha_i^L} P_m{}^{\alpha_i^M} Y_i$
$P_U U - P_X^{\beta_X} P_Y^{\beta_Y} U = 0$
$P_{U^G} U^G - P_X^{\beta_X^G} P_Y^{\beta_Y^G} P_L^{\beta_L} U^G = 0$
Market clearance condition
$\overline{K}_i = \widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i} P_i Y_i$
$\frac{\overline{L}}{(1-\overline{Un})} - \left(\frac{\overline{L}}{(1-\overline{Un})}\right) Un = \sum_{i=1}^{3} \frac{\alpha_i^L}{w} P_i Y_i$
$Y_i = C_i + G_i$
$C_X = \frac{\beta_X}{P_X} M + X$
$C_Y = \frac{\beta_Y}{P_Y} M = Y$
$G_X = \frac{\beta_X^G}{P_X} M^G$
$G_Y = \frac{\beta_T^G}{P_Y} M^G$
$Un(w^{min} - P_U) = 0$
$w = (P_{Ln} + \tau)$
Income constraint
$M = \sum_{j=A}^{B} r_j \widehat{K}_j + P_{Ln}\left(\frac{\overline{L}}{(1-\overline{Un})}\right) - P_{Ln}\left(\frac{\overline{L}}{(1-\overline{Un})}\right) Un + sub\,\overline{Transfers} = \sum_{i=A}^{B} P_i C_i$; being $\overline{K}_\iota = \sum_{i=1}^{n} \widehat{K}_i$; and $P_{Ln} = w - \tau$
$M^G = taxes - sub\,\overline{Transfers} = \sum_{i=A}^{B} P_i G_i$

7.4.4 Open economy with involuntary unemployment and unemployment benefits

The model relies on the same assumptions as those in the open economy with voluntary unemployment. The model is shown in Table 4.

Table 4. The equations of the small open economy CGE model with involuntary unemployment and unemployment benefits.

Zero profit
$(Pe_X E_X) - (P_X E_X) = 0$
$P_i Y_i - r_i^{\alpha_i^K} w^{\alpha_i^L} {P_m}^{\alpha_i^M} Y_i = 0$
$(wL_s) - (P_L L_s) = 0$
$P_U U - P_X^{\beta_X} P_Y^{\beta_Y} U = 0$
$P_G U^G - P_X^{\beta_X^G} P_Y^{\beta_Y^G} U = 0$
Market clearance condition
$\bar{K}_i = \hat{K}_i = \frac{\alpha_i^{\hat{K}}}{r_i} P_i Y_i$
$\left(\frac{\bar{L}}{(1-\overline{Un})}\right) - \left(\frac{\bar{L}}{(1-\overline{Un})}\right) Un = L_i = \frac{\alpha_i^L}{w} P_i Y_i$
$\bar{L} = \frac{1}{P_L} w L_s + \frac{\beta_L}{L} M$
$Y_X = \frac{1}{P_X} Pe_X E_X$
$m_i = \frac{\alpha_i^M}{Pm} P_i Y_i$
$C_X = \frac{\beta_X}{P_X} M = X$
$C_Y = \frac{\beta_Y}{P_Y} M = Y$
$G_X = \frac{\beta_X^G}{P_X} M^G$
$G_Y = \frac{\beta_T^G}{P_Y} M^G$
$Y_Y = C_Y - m_Y$
$Y_X = C_X + E_Y - m_X.$
$Pe = er\, Pe^w$
$Pm = er\, {P_m}^w$
$E_Y - m_Y - m_X = ca$
$Un\left(w^{min} - P_U\right) = 0$
$w = (P_{Ln} + \tau)$
Income constraint
$M = \sum_{j=A}^{B} r_j \hat{K}_j + P_{Ln}\left(\frac{\bar{L}}{(1-\overline{Un})}\right) - P_{Ln}\left(\frac{\bar{L}}{(1-\overline{Un})}\right) Un + sub\,\overline{Transfers} + ca = \sum_{i=A}^{B} P_i C_i$; being $\bar{K}_\iota = \sum_{i=1}^{n} \hat{K}_i$
$M^G = taxes - sub\,\overline{Transfers} = \sum_{i=A}^{B} P_i G_i$

7.4.5 Derived demand

The model assumes a new good/sector, called Y_Z, the production of which is entirely demanded by the other two sectors (Y_X and Y_Y) as input. Both $\bar{K}_\iota$ and $\bar{L}$ are supplied perfectly inelastically to the factor markets.

Sectoral behaviour

$$\text{(1.M3)}\ \ max_{Y_i,Z_i,L_i,\widehat{K}_i}\ (P_iY_i) - \left(P_ZZ_i + r_i\widehat{K}_i + wL_i\right)$$

$$subject\ to\colon Y_i = f(Z_i, \widehat{K}_i, L_i)$$

where Z_i denotes the new input demanded by the i sectors, and P_Z the input price. As noted, sector Z does not demand intermediate inputs, but labour and capital. Hence, when $j = Z$, the previous maximizing problem reduces to:

$$\text{(2.M3)}\ \ max_{Y_z,L_z,\widehat{K}_z}\ (P_zY_z) - \left(r_z\widehat{K}_z + wL_z\right)$$

$$subject\ to\colon Y_Z = f(Z_Z, \widehat{K}_Z, L_Z)$$

The respective demand functions of the previous problems are: $\widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i}P_iY_i$; $L_i = \frac{\alpha_i^L}{w}P_iY_i$; $Z_i = \frac{\alpha_i^Z}{P_z}P_iY_i$; $\widehat{K}_Z = \frac{\alpha_Z^{\widehat{K}}}{r_Z}P_ZY_Z$; $L_Z = \frac{\alpha_Z^L}{w}P_ZY_Z$. The zero-profit conditions are: P_iY_i-$r_i^{\alpha_i^K}w^{\alpha_i^L}P_Z^{\alpha_i^K}Y_i = 0$; and P_zY_Z-$r_z^{\alpha_z^K}w^{\alpha_z^L}Y_z = 0$. The other model assumptions and equations remain the same as stated in the example of voluntary unemployment. The model with derived demand is summarized in Table 5.

Household behaviour

$$\text{(3.M3)}\ \ max_{C_X,C_Y}\ U(C_X, C_Y)$$

$$subject\ to\colon M = \sum\nolimits_{i=A}^{B} P_iC_i$$

Household income constraint, denoted by equation (9.M1), is also accommodated as: $M = \sum_{j=A}^{B} r_j\widehat{K}_j + w\bar{L}$; (9.M3), where w refers to the wage and r_j the price of capital in each sector. The representative household continues to operate with a Cobb-Douglas utility function ($U = C_X^{\beta_X}C_Y^{\beta_Y}$). Hence the demand functions of C_X and C_Y are: $C_X = \frac{\beta_X}{P_X}M$; $C_Y = \frac{\beta_Y}{P_Y}M$. In sum, the zero-profit condition is: $P_UU - P_X^{\beta_X}P_Y^{\beta_Y}U = 0$. Finally, the equations of the model are shown in Table 5.

Table 5. The equations of the CGE model with derived demand.

Zero profit
$P_iY_i - r_i^{\alpha_i^K} w^{\alpha_i^L} P_Z^{\alpha_i^Z} Y_i = 0$
$P_zY_Z - r_z^{\alpha_z^K} w^{\alpha_z^L} Y_z = 0$
$P_U U - P_X^{\beta_X} P_Y^{\beta_Y} U = 0$
Market clearance condition
$\overline{K}_i = \widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i} P_i Y_i$
$\bar{L} = \sum_{i=1}^{3} \frac{\alpha_i^L}{w} P_i Y_i$
$Z = \sum_{i=1}^{2} \frac{\alpha_i^Z}{P_Z} P_i Y_i$
$C_X = \frac{\beta_X}{P_X} M = X$
$C_Y = \frac{\beta_Y}{P_Y} M = Y$
$Y_X + Y_Y = C_X + C_Y$
Income constraint
$M = \sum_{i=1}^{n} r_i \widehat{K}_i + w\bar{L} = \sum_{i=A}^{B} P_i C_i$; being $\overline{K_\iota} = \sum_{i=1}^{n} \widehat{K}_i$

7.4.6 Open economy with derived demand

The model relies on the same assumptions as those in the open economy with voluntary unemployment, but assumes a perfectly elastic labour supply. The model is shown in Table 6.

Table 6. The equations of the small open economy CGE model with derived demand

Zero profit
$(Pe_X E_X) - (P_X E_X)=0$
$P_i Y_i$- $r_i^{\alpha_i^K} w^{\alpha_i^L} Pm^{\alpha_i^M} P_Z^{\alpha_i^Z} Y_i = 0$
$P_z Y_Z$- $r_z^{\alpha_z^K} w^{\alpha_z^L} Y_z = 0$
$P_U U - P_X^{\beta_X} P_Y^{\beta_Y} U = 0$
Market clearance condition
$\bar{K}_i = \hat{K}_i = \frac{\alpha_i^{\hat{K}}}{r_i} P_i Y_i$
$\bar{L} = \sum_{i=1}^{3} \frac{\alpha_i^L}{w} P_i Y_i$
$Y_Z = \sum_{i=1}^{2} \frac{\alpha_i^Z}{P_Z} P_i Y_i$
$Y_Y = \frac{1}{P_Y} Pe_Y E_Y$
$m_i = \frac{\alpha_i^M}{Pm} P_i Y_i$
$C_X = \frac{\beta_X}{P_X} M = X$
$C_Y = \frac{\beta_Y}{P_Y} M = Y$
$Y_Y = C_Y - m_Y$
$Y_X = C_X + E_X - m_X$.
$Pe = er\ Pe^w$
$Pm = er\ Pm^w$
$E_Y - m_Y - m_X = ca$
Income constraint
$M = \sum_{j=A}^{B} r_j \hat{K}_j + w\bar{L} = \sum_{i=A}^{B} P_i C_i$; being $\bar{K_\iota} = \sum_{i=1}^{n} \hat{K}_i$

7.4.7 Derived demand with involuntary unemployment without unemployment benefits

This model aims to test the role of idle resources (involuntary unemployment in this case) when addressing projects with derived demand. Intuitively, when assuming full use of resources, an improvement in the cost of this factor immediately affects the other sectors that benefit from demanding a cheaper input. However, when assuming involuntary unemployment, the same reduction in cost in this input may enhance a second effect by allowing the other sectors to increase their demand for labour, thereby achieving a higher social welfare variation, though for a net welfare effect of the project this variation may be irrelevant when it is approximately common to the next best alternative. The model maintains the same structure but without unemployment benefits. Hence, the income constraint stands now as: $M = \sum_{j=A}^{B} r_j\widehat{K}_j + P_{Ln}\left(\frac{\bar{L}}{(1-\overline{Un})}\right) - P_{Ln}\left(\frac{\bar{L}}{(1-\overline{Un})}\right)Un = \sum_{i=A}^{B} P_iC_i$. The model's remaining equations are shown in Table 7.

Table 7. The CGE model's equations with derived demand and involuntary unemployment

Zero profit
P_iY_i- $r_i^{\alpha_i^K} w^{\alpha_i^L} P_Z^{\alpha_i^Z} Y_i = 0$
P_zY_Z- $r_z^{\alpha_z^K} w^{\alpha_z^L} Y_z = 0$
$P_U U - P_X^{\beta_X} P_Y^{\beta_Y} U = 0$
Market clearance condition
$\overline{K}_i = \widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i} P_iY_i$
$\left(\frac{\bar{L}}{(1-\overline{Un})}\right) - \left(\frac{\bar{L}}{(1-\overline{Un})}\right)Un = \sum_{i=1}^{3} \frac{\alpha_i^L}{w} P_iY_i$
$Z = \sum_{i=1}^{2} \frac{\alpha_i^Z}{P_Z} P_iY_i$
$C_X = \frac{\beta_X}{P_X} M = X$
$C_Y = \frac{\beta_Y}{P_Y} M = Y$
$Y_X + Y_X = C_X + C_Y$
Income constraint
$M = \sum_{j=A}^{B} r_j\widehat{K}_j + P_{Ln}\left(\frac{\bar{L}}{(1-\overline{Un})}\right) - P_{Ln}\left(\frac{\bar{L}}{(1-\overline{Un})}\right)Un = \sum_{i=A}^{B} P_iC_i$.; being $\overline{K}_i = \sum_{i=1}^{n} \widehat{K}_i$

7.4.8 Derived demand with a negative externality

As stated, a sector that produces with an externality is producing with a marginal cost lower than the social cost, causing a provision of this good above what is socially desirable, which implies greater welfare variation with respect to a situation without the externality. Specifically, the externality is modelled assuming the following production function: $Y_Y = Y_Y^{1-\beta} F(K, L)$, where $F(K, L)$ shows constant returns to scale. Or, alternatively, $Y_Y = F(K, L)^{\frac{1}{1-\beta}}$, where Y is homogenous of degree $\frac{1}{1-\beta} > 1$. Hence, the factor Y^{β} operates as the externality, causing greater production of good Y. The optimal demands of K and L of this sector are now: $pY^{\beta} \frac{\partial Y_Y}{\partial L} = w$ and $pY^{\beta} \frac{\partial Y_Y}{\partial K} = r$.

This case is an extension of the model with derived demand, but omitting the involuntary unemployment, and assuming that one sector (sector Y_Y) produces with an externality. Table 8 summarizes the model's equations. Instead of analyzing the impact of an externality solely, the idea is to combine both economic situations, intermediate demand and externalities, in order to provide a more comprehensive approach.

Table 8. The CGE model's equations with derived demand and a negative externality.

Zero profit
$P_x Y_x$- $r_x^{\alpha_i^K} w^{\alpha_x^L} P_Z^{\alpha_x^Z} Y_x = 0$ $P_y Y_y$- $r_y^{\alpha_i^K} w^{\alpha_y^L} P_Z^{\alpha_x^Z} Y_y \frac{1}{Y^\beta} = 0$ $P_z Y_Z$- $r_z^{\alpha_z^K} w^{\alpha_z^L} Y_z = 0$ $P_U U - P_X^{\beta_X} P_Y^{\beta_Y} U = 0$
Market clearance condition
$\overline{K}_i = \widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i} P_i Y_i,$ where $i = Y_x \ and \ Y_z$ $\overline{K}_Y = \widehat{K}_Y = \frac{\alpha_Y^{\widehat{K}}}{r_Y} P_Y Y_Y Y_y^{1-\beta}$ $\overline{K}_i = \widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i} P_i Y_i$ $\overline{L} = \frac{\alpha_x^L}{w} P_x Y_x + \frac{\alpha_z^L}{w} P_z Y_z + \frac{\alpha_y^L}{w} P_y Y_y Y_y^{1-\beta} = 0$ $Z = \sum_{i=1}^{2} \frac{\alpha_i^Z}{P_Z} P_i Y_i,$ where $i = Y_x \ and \ Y_y$ $C_X = \frac{\beta_X}{P_X} M = X$ $C_Y = \frac{\beta_Y}{P_Y} M = Y$ $Y_X + Y_Y = C_X + C_Y$
Income constraint
$M = \sum_{j=A}^{B} r_j \widehat{K}_j + w\overline{L} = \sum_{i=A}^{B} P_i C_i$.; being $\overline{K_\iota} = \sum_{i=1}^{n} \widehat{K}_i$; where $i = Y_x, Y_y \ and \ Y_z$

7.4.9 Derived demand and non-competitive markets

Finally, the analysis is extended to address the impact of assuming non-competitive behaviour in one market for the CGE model with derived demand. Specifically, the model assumes that one of the output markets (Y_Y) operates in a monopolistic market. The remaining assumptions and structure of the model resemble the CGE model with derived demand. The model is shown in Table 9. The variable $Markup$ is the benefit of the monopoly and causes the price to be higher than its marginal cost: $P_Y > MC_Y$. At the same time, the $Markup$ depends on the elasticity of substitution (σ) and on the share of the expenditure of the representative households on this good (Sh_Y). Finally, Table 10 summarizes each CGE model's main assumptions.

Table 9. The equations of the small open economy CGE model with derived demand and a non-competitive market.

Zero profit
$Markup P_Y Y_Y - r_i^{\alpha_Y^K} w^{\alpha_Y^L} P_Z^{\alpha_Y^Z} P_Y Y_Y = 0$
$P_X Y_X - r_i^{\alpha_X^K} w^{\alpha_X^L} P_Z^{\alpha_X^Z} P_X Y_X = 0$
$P_z Y_Z - r_z^{\alpha_z^K} w^{\alpha_z^L} Y_z = 0$
$P_U U - P_X^{\beta_X} P_Y^{\beta_Y} U = 0$
Market clearance condition
$\overline{K}_i = \widehat{K}_i = \frac{\alpha_i^{\widehat{K}}}{r_i} P_i Y_i$
$\overline{L} = \sum_{i=1}^{3} \frac{\alpha_i^L}{w} P_i Y_i$
$Z = \sum_{i=1}^{2} \frac{\alpha_i^Z}{P_Z} P_i Y_i$
$C_X = \frac{\beta_X}{P_X} M = X$
$C_Y = \frac{\beta_Y}{P_Y} M = Y$
$Y_X + Y_Y = C_X + C_Y$
Income constraint
$M = \sum_{i=1}^{n} r_i \widehat{K}_i + w\overline{L} + Markup = \sum_{i=A}^{B} P_i C_i$; being $\overline{K_\iota} = \sum_{i=1}^{n} \widehat{K}_i$
$Markup = 1/(\sigma - (\sigma - 1) Sh_Y)$
$Sh_Y = P_Y Y_Y / (P_Y Y_Y + P_X Y_X)$

Table 10. Overview of each CGE model.

	CGE models with voluntary unemployment	CGE models with involuntary unemployment and unemployment benefits	CGE models with derived demand	CGE models with derived demand with involuntary unemployment without unemployment benefits	CGE models with derived demand and a negative externality	CGE models with derived demand and a non-competitive market
Closed economy	Two sectors, two factors (capital and labour), one representative household.	Two sectors, two factors (capital and labour), one representative household, one government.	Three sectors, three factors (capital, labour and intermediate demand). The output of one of the sectors is demanded as input by the other two sectors. One representative household.	Three sectors, three factors (capital, labour and intermediate demand). The output of one of the sectors is demanded as input by the other two sectors. One representative household.	Three sectors, three factors (capital, labour and intermediate demand). The output of one of the sectors is demanded as input by the other two sectors. One representative household.	Three sectors, three factors (capital, labour and intermediate demand). The output of one of the sectors is demanded as input by the other two sectors. One representative household.
Open economy	Two sectors, three factors (capital, labour and imports), one representative household. Only one sector exports abroad (the tradable sector).	Two sectors, three factors (capital, labour and imports), one representative household. Only one sector exports abroad (the tradable sector).	Three sectors, three factors (capital, labour and imports), one representative household. Only one sector exports abroad (the tradable sector).	-	-	-

7.4.10 Parameter calibrations and shocks

The parameters employed in the calibration of the models are shown in Table 11. The shocks simulated in each model aim at triggering the economic effects conducted by CBA. Specifically, the shocks assumed in the models with voluntary unemployment and involuntary unemployment represent an increase in capital productivity in sector Y_Y; whereas the shock assumed in the model with derived demand represents an increase in total factor productivity (capital and labour) in sector Y_Z [13]. All models have been programmed in GAMS using MPSGE (Rutherford, 1999).

[13] See Annex I for a stylized formal demonstration of the economic impact of this shock.

Table 11. Calibrated parameters

	Voluntary unemployment		Involuntary unemployment		Derived demand	
	Closed economy	Open economy	Closed economy	Open economy	Closed economy	Open economy
α_X^K	0.6	0.5	0.6	0.50	0.41	0.25
α_X^L	0.4	0.33	0.4	0.28	0.26	0.22
α_X^M	-	0.17	-	0.14	-	0.16
α_X^Z	-	-	-	-	0.33	0.37
α_Y^K	0.4	0.33	0.4	0.24	0.26	0.25
α_Y^L	0.6	0.5	0.6	0.5	0.41	0.37
α_Y^M	-	0.17	-	0.26	-	0.12
α_Y^Z	-	-	-	-	0.33	0.25
α_Z^L	-	-	-	-	0.6	0.6
α_Z^K	-	-	-	-	0.4	0.4
β_X	0.5	0.31	0.47	0.47	0.5	0.5
β_Y	0.5	0.37	0.53	0.53	0.5	0.5
β_L	0.5	0.31	-	-	0.5	-
β_X^G	-	-	0.8	0.8	-	-
β_Y^G	-	-	0.2	0.2	-	-
M	300	320	210	230	300	320
M^G	-	-	10	10	-	-
Un	-	-	0.2	0.2	-	-
Sub	-	-	0.2	0.2	-	-
τ	-	-	0.2	0.2	-	-

7.5 Results from the CGE models

CGE models form a system of simultaneous equations with n equations and $n + 1$ variables. Fortunately, all the equations are homogenous of degree 1 in prices. Thus, a model of this kind allows us to fix one variable to unity. This variable is known as the *numeraire,* and hence, all prices are interpreted in relative terms. In our case, the *numeraire* chosen is P_U (the consumer price index) because it allows for a more intuitive interpretation of the other prices in the economy (in real terms). Similarly, the variables that represent quantities equal one in the initial equilibrium. Hence, they also must be interpreted in relative terms (i.e., suppose the production of good Y_X grows from 1 to 1.01, this means that the initial production has been multiplied by 1.01, or that the new output is 1% higher than in the initial equilibrium)[14]. On the other hand, the income level and all welfare measures used throughout this Section are shown in absolute values. Specifically, the analysis distinguishes two welfare measures: the equivalent variation obtained through the CGE approach (SW^{CGE}), and the IWA.

7.5.1 Voluntary unemployment

As shown in Table 12, an increase in capital productivity (5%) when assuming voluntary unemployment causes an increase in production in the *primary* market (Y_Y) of 1.029. This additional production implies an increase in the demand for labour that pushes up wages, which triggers two additional effects. On the one hand, taking into account that the model assumes perfect labour mobility, the *secondary* market (Y_X) cannot afford to pay the higher wages, which causes a reduction in production (0.998) and labour displacement in favour of the primary market. Employment decreases from 40 to 39.757 in this market. On the other, the demand for labour in the *primary* market also creates new employment as captured by the $Labour\ supply$ that rises from 100 to 100.609. This new employment (0.609) is created at the cost of reducing $Leisure$ by the same magnitude (100-99.391) (i.e., the higher wages increase the opportunity cost of leisure (P_{Le}), which is now 1.013; fostering the labour supply). In sum, when assuming voluntary unemployment, the labour market behaves as conducted by CBA. Finally, the results show a social welfare gain (2.004) when analyzing the total equivalent variation (SW^{CGE}).

[14] For further information, see Hosoe, *et al*. (2010).

Table 12. Results of the model with voluntary unemployment (5% shock).

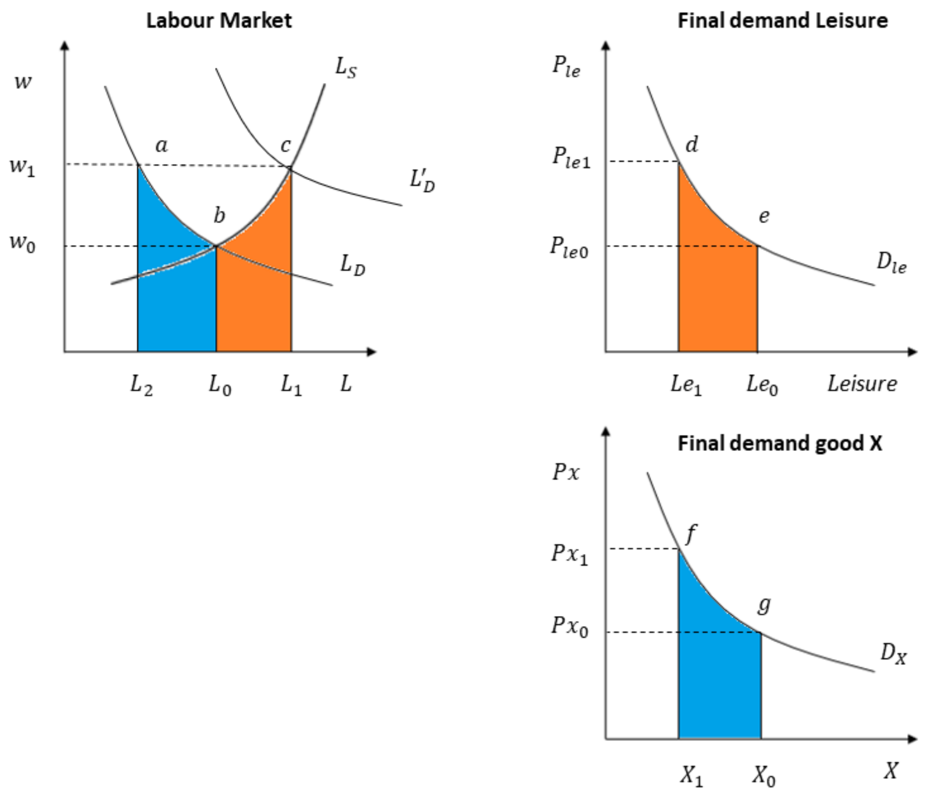
	Initial equilibrium	Final equilibrium
Y_X	1	0.998
Y_Y	1	1.029
T	1	1.006
P_X	1	1.007
P_Y	1	0.978
P_L	1	1.013
w	1	1.013
r_X	1	1.007
r_Y		0.976
$P_U(numeraire)$	1	1
$Income$	300	302.004
SW^{CGE}	-	2.004
$Employment\ X$	40	39.757
$Employment\ Y$	60	60.852
$Total\ Employment$	100	100609
$Leisure$	100	99.391
$Labour\ supply$	100	100.609
ALD	-	-0.244
ALC	-	0.607
ΔS_X	-	-0.244
$\Delta S_{Leisure}$	-	-0.607
IWA	-	2.004

Returning to Figure 1, the area abL_0L_2 represents the value of the production lost in the *secondary* market because of the displacement of labour from the former to the *primary* market. Similarly, the area bcL_1L_0 represents society's opportunity cost (the cost of leisure) of hiring these extra workers because of the project (voluntary unemployed, previous to the project). These areas are reported in Table 12 and Figure 4. They are, respectively: ALD (Area of Labour Displaced, blue-coloured area) and ALC

(Area of Labour Created, orange-coloured area), which, at the same time, coincide with the change observed in their respective final demands approached by the respective total surpluses: ΔS_X for good X (blue-coloured area) and $\Delta S_{Leisure}$ for leisure (orange-coloured area).

In sum, all changes (opportunity costs) triggered by the project in a labour market with voluntary unemployment are correctly included in the final demand of the representative household in a CGE framework (i.e., the welfare change of a project can be approached by merely concentrating on the representative agent, as is generally done by CGE). Alternatively, the welfare change can also be approached by focusing on the changes observed in the income (Income Welfare Approach, IWA) (IWA=SW^{CGE}).

Figure 4. Equivalence between the labour market's opportunity costs and final demands.



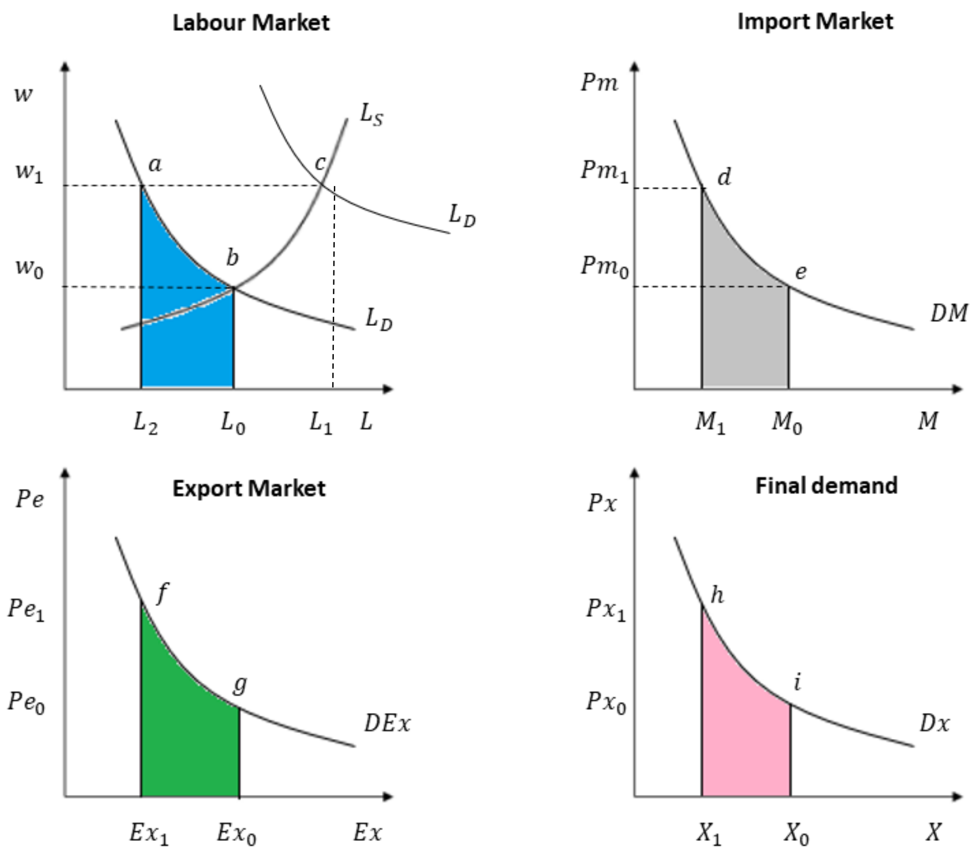
The identity between the opportunity cost of leisure and the area of labour created holds when assuming a small-open economy setting (see Table 13). However, the analysis of the labour force displaced from good Y_X (ALD) and its equivalence in the final demand of this good has to be extended to deal with other factors. On the one hand, the production of Y_X now requires demanding imports. On the other, good Y_X can also be exported. As a result, variation in the total surplus of this good (ΔS_X) equals the variation in labour demand (ALD), the variation in imports demand (AMD) and the variation in exports (AXD), such as $\Delta S_x = ALD + AMD - AXD$. Finally, ARD denotes the variation of factors displaced from sector Y_X that equates ΔS_x.

Table 13. A small open economy with voluntary unemployment (5% shock).

	Zero deficit		**Deficit**		**Surplus**	
	Initial equilibrium	**Final equilibrium**	**Initial equilibrium**	**Final equilibrium**	**Initial equilibrium**	**Final equilibrium**
E_X	1	0.983	1	1.012	1	1.004
Y_X	1	0.994	1	0.999	1	0.999
Y_Y	1	1.029	1	1.025	1	1.024
T	1	1.004	1	1.006	1	1.006
P_X	1	1.011	1	1.010	1	1.012
P_Y	1	0.978	1	0.982	1	0.983
P_L	1	1.011	1	1.012	1	1.013
w	1	1.011	1	1.012	1	1.013
r_X	1	1.011	1	1.009	1	1.012
r_Y	1	0.978	1	0.972	1	0.973
rer *(real exchange rate)*	1	1.011	1	1.010	1	1.012
$P_U(numeraire)$	1	1	1	1	1	1
$Income$	300	302.007	320	321.996	280	281.997
SW^{CGE}	-	2.007	-	1.996	-	1.997
$Employment\ X$	40	39.352	40	39.881	40	39.938
$Employment\ Y$	60	61.088	60	60.677	60	60.655
$Total\ Employment$	100	100.440	100	100.558	100	100.593
$Leisure$	100	99.560	100	99.442	100	99.913
$Labour\ supply$	100	100.440	100	100.558	100	100.086
ALC	-	0.44	-	0.55	-	0.59
$\Delta S_{Leisure}$	-	-0.44	-	-0.55	-	-0.59
ALD	-	-0.654	-	-0.11	-	-0.062
ΔY_{M_X}		-0.130		-0.024		-0.012
ΔY_{E_X}	-	-0.351	-	0.236	-	0.224
ARD		-0.43	-	-0.38		-0.30
ΔS_X	-	-0.43	-	-0.38	-	-0.30
IWA	-	2.007	-	1.996	-	1.997

Figure 5 shows this new equivalence when assuming a zero deficit in the current account position. ΔS_x (Pink-coloured area) = ALD (blue-coloured area) + AMD (grey-coloured area) – AXD (green-coloured area).

Figure 5. Equivalence between the opportunity costs in the input market and final demands of *X* in an open economy setting, with zero deficit.



In sum, CGE models are capable of modelling voluntary unemployment and capturing all the opportunity costs that take place in this market, as noted by CBA. Further, changes observed in the labour market are implicitly included in the total equivalent variation

SW^{CGE}). Thus, from a CGE perspective, the welfare changes that take place in the labour market are directly observed in the final demand (representative household). The results hold when assuming a small open economy setting.

7.5.2 Involuntary unemployment and unemployment benefits

In economic terms, the existence of involuntary unemployment (idle workforce) implies a different *model closure* compared with those previous. The underlying idea is that the model is in equilibrium with all markets clearing, except for the labour market that operates with an excess of supply. In terms of the model's adjustment, it implies that the economy may eventually grow without diverting labour from other activities or uses; causing a different economic adjustment, as explained below.

Table 14. Results of the model with involuntary unemployment and unemployment benefits (5% shock).

	Initial equilibrium	Final equilibrium
Y_X	1	1.018
Y_Y	1	1.048
U	1	1.023
U^G	1	1.244
P_X	1	1.023
P_Y	1	0.980
w	1	1
r_X	1	1.041
r_Y	1	0.993
$P_U (numeraire)$	1	1
$Income$	210	241.817
SW^{CGE}	-	1.475
IWA	-	1.475
ΔS_X	-	0.013
ΔS_Y	-	4.715
$Unemployment$	0.2	0.165
$Subsidies$	0.2	0.165
$Employment\ X$	40	41.657
$Employment\ Y$	60	62.706

Table 14 shows the results of a 5% shock in capital productivity. The first difference, when compared with previous models, is that, both sectors (Y_X and Y_Y) increase their production (by 1.018 and 1.048, respectively). Specifically, this is caused by the involuntary unemployment that allows all sectors to increase their output by demanding more workers. As a result, the unemployment rate falls from 0.2 to 0.165. This *model closure* also affects the adjustment in other factors of production (capital). This resource keeps clearing at the market price, is still a sector-specific factor, and is also supplied perfectly inelastically to the market (fixed supply).

Nonetheless, the increase in production in both sectors forces an increase in the price of this input. In other words, involuntary unemployment triggers a double induced effect. On the one hand, there are more workers employed, which at the same time means lower unemployment benefits (lower public expenses). On the other, capital is also more demanded, but its supply is fixed; and hence, its price increases. Thus, the $Income$ increases from 210 to 241.817. The SW^{CGE} continues to equate with the change in income (IWA), measured by the 'change in the income constraint' (1.475). In this sense, income constraint has been enriched to fulfil two additional roles to reconcile CBA and CGE. Firstly, wages enter net of taxes in income constraint. Secondly, it takes into account the reduction in unemployment benefits that is now retained by the government. Hence, in the context of involuntary unemployment, the income constraint is also capable of measuring the labour opportunity cost by subtracting income labour taxes and unemployment benefits, as postulated by CBA (Johansson and Kriström, 2022), and shown in Figure 2.

Under an open economy framework (see Table 15), the economic impacts of the project result in a real exchange depreciation (rer) that increase exports (E_X). The remaining results are similar to those of the closed economy. In terms of welfare, the conclusions are the same. The SW^{CGE} continues to equate to the IWA.

Table 15. A small open economy with involuntary unemployment and unemployment benefits (5% shock).

	Zero deficit	Deficit	Surplus
E_X	1.013	1.024	1.010
Y_X	1.013	1.011	1.015
Y_Y	1.034	1.032	1.036
U	1.007	1.015	1.020
U^G	1.177	1.161	1.2
P_X	1.019	1.016	1.022
P_Y	0.986	0.985	0.987
w	1	1	1
r_X	1.032	1.028	1.037
r_Y	0.980	0.978	0.984
rer	1.019	1.016	1.022
$P_U(numeraire)$	1	1	1
$Income$	213.561	233.341	193.856
SW^{CGE}	3.561	3.341	2.497
IWA	3.561	3.341	2.497
$Unemployment$	0.174	0.177	0.171
$Subsidies$	0.174	0.177	0.171
$Employment\ X$	41.275	41.116	41.485
$Employment\ Y$	61.920	61.758	62.144

7.5.3 Derived demand

As shown in Table 16, an increase in total factor productivity in the input market (Y_Z) reduces its cost of production (0.966), causing an increase in output (1.053) (direct effect). Similarly, taking into account that this good is demanded as an intermediate good by the output markets (Y_X and Y_Y), both are capable of increasing its production as well (1.017 and 1.017, respectively). Finally, the shock also increases wages (w) and the remuneration of capital (r_X and r_Y) triggering an induced effect as captured by the rise in $Income$. In terms of welfare change, Table 16 now reports the variation in total surplus (ΔS), both in the input market (ΔS^{input}) and the output markets (ΔS^{output}). The results show that $\Delta S^{input} = \Delta S^{output}$, meaning that, as higlighted by CBA in the case of derived demand, the welfare analysis in CGE can also focus on the input market/s or on the output markets. It should be remembered that ΔS^{output} differs from SW^{CGE} because the latter is approached by the equivalent variation. Finally, the input market multiplicative effect (Pme) is introduced into the analysis. As shown, its value coincides with the welfare change of the representative household (U), meaning that the 5% increase in total factor productivity boosts a multiplicative effect in sector Y_Z, which coincides with the multiplicative change in total welfare (SW^{CGE}). More precisely, the conclusion holds regardless of the magnitude of the project, as shown in Table 17.

Table 16. Results of the model with derived demand (5% shock).

	Initial equilibrium	Final equilibrium
Y_X	1	1.017
Y_Y	1	1.017
Y_Z	1	1.053
P_X	1	1.000
P_Y	1	1.000
P_Z	1	0.966
U	1	1.017
w	1	1.01 7
r_X	1	1.017
r_Y	1	1.017
r_Z	1	1.017
$P_U(numeraire)$	1	1
$Income$	300	305.173
SW^{CGE}	1	5.173
ΔS^{input}	1	5.129
ΔS^{output}	1	5.129
IWA	1	5.173
Pme	1	1.017

Table 17. Results of the model with derived demand by varying the magnitude of the shock

	Final equilibrium (1% shock)	Final equilibrium (5% shock)	Final equilibrium (10% shock)
Y_X	1.003	1.017	1.036
Y_Y	1.003	1.017	1.036
Y_Z	1.010	1.053	1.111
P_X	1.000	1.000	1.000
P_Y	1.000	1.000	1.000
P_Z	0.993	0.966	0.932
U	1.003	1.017	1.036
w	1.003	1.017	1.036
r_X	1.003	1.017	1.036
r_Y	1.003	1.017	1.036
r_Z	1.003	1.017	1.036
$P_U(numeraire)$	1	1	1
$Income$	301.007	305.173	310.723
SW^{CGE}	1.007	5.173	10.723
ΔS^{input}	1.005	5.129	10.536
ΔS^{output}	1.005	5.129	10.536
IWA	1.007	5.173	10.723
Pme	1.003	1.017	1.036

Table 18. Results of the model with derived demand in an open economy

	Zero deficit		**Deficit**		**Surplus**	
	1% shock	**10% shock**	**1% shock**	**10% shock**	**1% shock**	**10% shock**
SW^{CGE}	1.007	10.723	1.007	10.719	1.007	10.700
ΔS^{input}	1.005	10.536	1.005	10.543	1.005	10.525
ΔS^{output}	1.005	10.536	1.007	10.553	1.006	10.550
IWA	1.007	4.010	1.007	10.719	1.007	10.700
U	1.003	1.036	1.003	1.033	1.003	1.033
Pme	1.003	1.036	1.003	1.035	1.003	1.031

Table 18 shows that approaching the welfare variation by focusing on the input market coincides with the total surplus when assuming zero deficit ($\Delta S^{input} = \Delta S^{output} = \Delta Y_Z$). Nevertheless, it slightly differs with respect to the other *closures* since the value of the discrepancy increases with the magnitude of the project. In terms of the economic impact, the adjustment is very similar in all three cases (zero deficit, deficit and surplus). It should be noted that the current account position causes the main difference. When assuming zero deficit, the change in the current account does not affect income constraint. But in the other two cases, these kinds of variations affect the position; by increasing/decreasing the deficit/surplus.

7.5.4 Derived demand with involuntary unemployment without unemployment benefits

As expected, the existence of involuntary unemployment without unemployment benefits, in the context of derived demand, enhances a second effect by facilitating an increase in labour demand that boosts the economy's income level (see Table 19). As a result, the welfare impact is larger than that without involuntary unemployment. Assuming a shock of 10% in total factor productivity allows us to appreciate the previously described effects more starkly.

Table 19. Results of the model with derived demand and with involuntary unemployment.

	Derived demand (10% shock)	Derived demand with involuntary unemployment (10% shock)
Y_X	1.036	1.073
Y_Y	1.036	1.084
Y_Z	1.111	1.162
P_X	1.000	1.005
P_Y	1.000	0.995
P_Z	0.932	0.928
U	1.036	1.078
w	1.036	1.078
r_X	1.036	1.078
r_Y	1.036	1.078
r_Z	1.036	1.078
$P_U(numeraire)$	1	1
$Income$	310.723	323.449
SW^{CGE}	10.723	23.449
ΔS^{input}	10.536	22.577
ΔS^{output}	10.536	15.052
IWA	10.723	23.449
Pme	1.036	1.078

7.5.5 Derived demand with a negative externality

The model assumes a 5% shock to appreciate more clearly the welfare variation triggered by the externality. As shown in Table 20, the existence of an externality fosters a higher economic and welfare change when comparing the SW^{CGE}, the Pme or the IWA. In all cases, the three values are higher when assuming externalities. Alternatively, the cost of the externality can be *endogenized* by levying a tax on the production of sector Y_Y. In this case, the SW^{CGE} and the IWA would report an unbiased result.

It should be stressed that, as shown in section 7.4.8, in the case of externalities, ΔS and ΔY_Z diverges such as: $\Delta S > \Delta Y_Z$ ($\Delta S^{output} > \Delta S^{input}$). Furthermore, as can also be appreciated, the variation in the production of Y_Z is the same in both cases (with and without externality), showing that the variation of the total surplus in the input market (ΔS^{input}) provides an unbiased welfare evaluation when externalities are present in output markets.

Table 20. Results of the model with derived demand and a negative externality.

	Without externality (5% shock)	With externality (5% shock)
Y_X	1.017	1.017
Y_Y	1.017	1.017
Y_Z	1.053	1.053
P_X	1.000	1.002
P_Y	1.000	0.998
P_Z	0.966	0.968
U	1.017	1.019
w	1.017	1.019
r_X	1.017	1.019
r_Y	1.017	1.019
r_Z	1.017	1.019
$P_U (numeraire)$	1	1
$Income$	305.173	305.829
SW^{CGE}	5.0173	5.826
ΔS^{output}	5.129	5.770
ΔS^{input}	5.129	5.129
IWA	5.173	5.826
Pme	1.017	1.019

7.5.6 Derived demand with non-competitive markets

As shown in Table 21, the welfare measures continue to work adequately according to the theory. SW^{CGE} equals IWA, and U equals Pme. In CBA, the variation in production in the input market ($\Delta Y_Z = \Delta S^{input}$) would show a biased result unless the imperfect market situation in sector $Y_Y$is accounted for. Hence, $\Delta S^{output} > \Delta S^{input}$. In these cases, the welfare change that takes place in the non-competitive market must be included to that obtained in the input market, as done in CBA. Fortunately, the latter is suitably captured in CGE by the SW^{CGE} and IWA when focusing on the output markets. The intuition behind this result is like that of the open economy situation, or when assuming involuntary unemployment.

Table 21. Results of the model with derived demand and a non-competitive market.

	Initial equilibrium	Final equilibrium (5% shock)
Y_X	1	1.020
Y_Y	1	1.017
Y_Z	1	1.019
P_X	1	1.260
P_Y	1.261	1.001
P_Z	1	0.967
U	1	1.019
w	1	1.018
r_X	1	1.018
r_Y	1	1.019
r_Z	1	1.018
$Markup$	0.522	0.522
Sh_Y	0.207	0.207
$P_U(numeraire)$	1	1
$Income$	313.891	319.777
SW^{CGE}	1	5.843
ΔS^{output}	1	5.541
ΔS^{input}	1	5.136
IWA	1	5.843
Pme	1	1.019

7.6 The relevance of the counterfactual and model closure in CGE welfare appraisal

A proper economic evaluation requires us to consider counterfactual scenarios in order to compare the project's social benefit that are triggered with a reasonable alternative use of the resources. In this sense, three kinds of counterfactual are usually employed in CBA (European Investment Bank, 2013): "Do nothing", "Do the minimum", "Do something (else)".

Furthermore, the development of an investment project distinguishes two stages, with each generating its own economic and welfare impact: Stage 1, also known as CAPEX (capital expenditure), comprises the investment phase (construction). In terms of CBA, this stage represents a social cost, but it may also trigger economic and welfare effects. Stage 2, also known as OPEX (operational expenditure), takes place once the infrastructure is implemented, and implies social changes in the welfare of the economy.

A CGE model will take into account all the multiplier effects of the investment phase. These are relevant if the economy is working with involuntary unemployment. In this scenario, the investment phase implies production, lower unemployment, and higher income that leads to higher consumption and firms earning higher profits. These are known as multiplier effects, which are a second-round income effect that happen in the economy after any income shock. This effect occurs in the whole economy and not necessarily in the project's markets of interest. However, the multiplier effect is not required to be measured in CBA when the counterfactual project is expected to impulse the multiplier effects in a similar way. In CGE, the multiplier effects are computationally always part of the results, so that, for an adequate comparison between CGE and CBA they need to be calculated within CGE, and deducted.

In order to deal with this issue, we have considered a counterfactual scenario consisting of returning the investment funds (lump sum transfer) to the taxpayers (representative household). Alternative scenarios may be considered, such as alternative investment projects. The idea is to compare both policies and to decide if the investment project, in its CAPEX phase, results in a welfare gain above the counterfactual scenario.

The model employed in this section is similar to that with derived demand and involuntary unemployment (without unemployment benefits) (see Table 7), but adding the investment (INV) and slightly reformulating the government's role. The former is now demanded by the representative household and government, and it is generated according to the following zero profit condition: $P_{INV}INV - P_X^{\vartheta_X}P_Y^{\vartheta_Y}INV = 0$ where P_{INV} denotes the investment price and, ϑ_X and ϑ_Y denotes the share of goods X and Y in the generation of the investment. Thus, the final goods X and Y are now demanded as consumption and as investment. Regarding the government, it now collects indirect taxes on goods, transfers incomes to the representative household, obtains income from the rent of capital, consumes final goods and invests in capital goods, as previously mentioned. Finally, the analysis will also show the consequences, in terms of welfare, of choosing different *closures*. Specifically, the analysis focuses on assuming three different closures concerning the government's decision to finance the investment, as explained below.

Turning to the model, the government's maximizing problem is similar to problem 2.M2 in the CGE model with derived demand and involuntary unemployment, but adding the investment by goods ((Inv_i^G) multiplied by their prices ($Pinv_i$), the rents of capital ($rK_X^G + rK_Y^G$) and where $deficit$ denotes the government's budgetary position that, in this case, is running a deficit. The new maximizing problem is:

$$max_{G,Inv^G}\ U^G(G, Inv^G)$$

$$subject\ to: M^G = \sum_{i=A}^{B} P_i(1 + tax_i)G_i + Pinv_iInv_i^G$$

where $M^G = taxes + rK_X^G + rK_Y^G + P_{UG}deficit$, $G = G_X + G_Y$ and $Inv^G = Inv_X^G + Inv_Y^G$, which represent the total level of consumption and investment of the government, respectively, and $taxes = \sum_{i=X}^{Y} tax_i$.

Similarly, the maximizing problem of the households must be adapted to include the investment decision. The latter is similar to problem 1.M2.

$$max_{C,Inv^H}\ U(C, Inv^H)$$

$$subject\ to: M^H = \sum_{i=A}^{B} P_i(1 + tax_i)C_i + Pinv_iInv_i^H$$

where $C = C_X + C_Y$ and $Inv^{GH} = Inv_X^H + Inv_Y^H$, which represent the total level of household consumption and investment, respectively.

The household income constraint, denoted by equation (9.M1), is also accommodated as follows: $M^H = \sum_{j=A}^{B} r_j\widehat{K}_j + P_{Ln}\left(\frac{\bar{L}}{(1-Un)}\right) - P_{Ln}\left(\frac{\bar{L}}{(1-Un)}\right)Un + P_{UG}transfers,$; where now P_{Ln}, wages net of taxes, equal the gross wage w because there are no unemployment benefits, Un and $Transfers$ are parameters denoting the initial unemployment rate and the social transfers that equate to the government's budgetary position ($deficit = transfers$). Specifically, the way the government decides to finance these transfers defines an important *closure* of the model with a direct impact on the simulation, as noted. If the government decides to assume a fixed deficit, ($\Delta deficit = 0$) then this implies that the social transfer remains fixed as well. Hence, the government will be conditioned by keeping the budgetary position fixed. Alternatively, it could opt to allow the budgetary position to vary ($\Delta deficit \neq 0$). In all cases, the *closure* will affect the governmental decision of consumption and investment and thus, will also affect the economic impact and welfare change. The closure of the governmental position is even more important in this case, because both the investment project and the assumed counterfactual scenario imply public investment. The different closures are accommodated by establishing a constraint, such that: $\Delta deficit = deficit_0 - public_{investment} = 0, > 0\ or < 0$; where $deficit_0$ denotes the initial governmental position and $public_{investment}$ the public funds required to carry out the investment. For the sake of exposition, we will run the simulations under the three different closures ($\Delta deficit = 0$, $\Delta deficit > 0, \Delta deficit < 0$) to highlight the different welfare results. However, in project appraisals, the choice of the closure will depend on the characteristics of the economy and the case study. It should be stressed that, in its current formulation, we are following an investment-driven *closure* for both agents, because the level of investment is determined endogenously[15]. Finally, the investment project to be simulated implies a public investment in good Y_Z. The magnitude of the investment represents around 1.25% of this economy's GDP. Alternatively, the counterfactual scenario implies transferring this amount of public funds to households. Table 22 summarizes the main equations of the model, while Table 23 shows the model's calibrated parameters and values.

[15] In order to follow a savings-driven *closure*, the investment level should be assumed exogenously by establishing an endowment in the income constraint of both agents.

Table 22. The CGE's equations for the counterfactual analysis and different government *closures*

Zero profit

$P_i(1+tax_i)Y_i$- $r_i^{\alpha_i^K} w^{\alpha_i^L} P_Z^{\alpha_i^Z} Y_i = 0$, where $\sum_{i=A}^{B} tax_i = taxes$

$P_z Y_Z$- $r_z^{\alpha_z^K} w^{\alpha_z^L} Y_z = 0$

$P_U U - P_X^{\beta_X} P_Y^{\beta_Y} U = 0$

$P_{UG} U^G - P_X^{\beta_X^G} P_Y^{\beta_X^G} U^G = 0$

$P_{INV} INV - P_X^{\vartheta_X} P_Y^{\vartheta_Y} INV = 0$

Market clearance condition

$\overline{K} = \sum_{j=A}^{B} \widehat{K}_j + K_X^G + K_Y^G$

$$\left(\frac{\overline{L}}{(1-Un)}\right) - \left(\frac{\overline{L}}{(1-Un)}\right) Un = \sum_{i=1}^{3} \frac{\alpha_i^L}{w} P_i Y_i$$

$$Z = \sum_{i=1}^{2} \frac{\alpha_i^Z}{P_Z} P_i Y_i$$

$C = C_X + C_Y$

$Inv^H = Inv_X^H + Inv_Y^H$

$G = G_X + G_Y$

$Inv^G = Inv_X^G + Inv_Y^G$

$Y + X = C + G + Inv^H + Inv^G$

Income constraint

$M^H = \sum_{j=A}^{B} r_j \widehat{K}_j + P_{Ln}\left(\frac{\overline{L}}{(1-Un)}\right) - P_{Ln}\left(\frac{\overline{L}}{(1-Un)}\right) Un + P_{UG} transfers =$

$\sum_{i=A}^{B} P_i C_i + Pinv_i Inv_i^H$.;

$$M^G = taxes + rK_X^G + rK_Y^G + P_{UG} deficit = \sum_{i=A}^{B} P_i G_i + Pinv_i Inv_i^G$$

additional constraint

$\Delta deficit = deficit_0 - public\_investment = 0, > 0 \; or < 0$

Table 23. Calibrated parameters for the counterfactual analysis and different governmental closures

α_X^K	0.60
α_X^L	0.18
α_X^Z	0.22
α_Y^K	0.25
α_Y^K	0.40
α_Y^Z	0.22
α_Z^L	0.38
α_Z^K	0.4
ϑ_X	0.5
ϑ_Y	0.5
β_X	0.5
β_Y	0.5
β_X^G	0.5
β_Y^G	0.5
M	370
M^G	74.444
Un	0.10
$deficit$	70
$taxes$	0.1

Table 24 shows the results of the analysis. As can be appreciated, the economic impact and welfare vary slightly with the governmental *closure*. The main source of change is caused by governmental behaviour, measured by its equivalent variation, $SW_{government}^{CGE}$, that varies from -4.169 to -5.776 when assuming that the investment is financed with more deficit ($\Delta deficit > 0$) or less deficit ($\Delta deficit < 0$), respectively. In any event, in all cases the investment project triggers larger economic and welfare impacts than the counterfactual scenario. Likewise, the former also reduces the unemployment rate, which decreases from 0.1 in the equilibrium, to 0.082 in all cases.

Table 24. Results of the model with different governmental *closures*

	$\Delta deficit = 0$		$\Delta deficit > 0$		$\Delta deficit < 0$	
	Investment project	Counterfactual	Investment project	Counterfactual	Investment project	Counterfactual
Y_X	1.019	1.000	1.019	1.000	1.019	1.000
Y_Y	1.021	1.000	1.021	1.000	1.021	1.000
Y_Z	1.061	1.000	1.061	1.000	1.061	1.000
P_X	1.001	1.000	1.001	1.000	1.001	1.000
P_Y	0.999	1.000	0.999	1.000	0.999	1.000
P_Z	0.961	1.000	0.961	1.000	0.961	1.000
U	1.016	1.014	1.016	1.014	1.016	1.014
U^G	1.038	0.922	1.049	0.944	1.029	0.922
INV	1.019	1.012	1.010	0.994	1.027	1.012
w	1.000	1.000	1.000	1.000	1.000	1.000
r_X	1.020	1.000	1.020	1.000	1.020	1.000
r_Y	1.020	1.000	1.020	1.000	1.020	1.000
r_Z	0.907	1.000	0.907	1.000	0.907	1.000
$P_U(numeraire)$	1	1	1	1	1	1
$Unemployment$	0.082	0.10	0.082	0.10	0.082	0.10
$SW^{CGE}_{households}$	5.947	5	5.947	5	5.947	5
$SW^{CGE}_{government}$	2.864	-5.776	-2.136	-4.169	7.530	-5.776
SW^{CGE}	8.811	-0.776	3.811	0.831	12.477	-0.776

7.7 Conclusions

This paper has shown that a project's net welfare change can be approached in CGE by differences in the economy's income constraint - with and without the project - by employing the *Income Welfare Approach*. This result coincides with Johansson (2022) when deriving general equilibrium cost-benefit rules, and shows that when considering the same economic situations, both CBA and CGE should provide identical welfare measures.

Overall, the paper shows that CGE models are already capturing most of the opportunity costs (shadow prices) emphasized by CBA. Specifically, all welfare variations that take place in the different markets are captured in the CGE's final demand goods. The analysis demonstrates that all the economic changes that occur in the primary markets of an economy are included in the final demand of the representative/s household/s in a CGE model. Thus, the welfare analysis can focus on final demands. In this case, both the Equivalent Variation (EV) and the Income Welfare Approach (IWA) provide identical and suitable approaches for measuring any welfare variation.

The paper has shown that there is a special case that takes place when omitting the impact on output markets, so that the welfare in CGE and CBA equates when dealing with derived demand ($\Delta S^{output} = \Delta S^{input}$). However, as also noted by CBA, this result does not hold in certain open economy situations; or when assuming idle resources (involuntary unemployment), imperfect markets or externalities. For instance, the existence of involuntary unemployment with derived demand enhances a second effect by facilitating an increase in labour demand. As a result, the welfare impact is greater than without the assumption of involuntary unemployment. In any case, CBA does incorporate the value of distortions in any market when calculating a project's net welfare effect. Nevertheless, when the value of the distortions is related to the multiplier effect of the project, and the multiplier effect of the next best alternative is expected to be similar to the counterfactual, they can be ignored.

Similarly, in the case of derived demand with a negative externality, the welfare valuation when focusing on final demand, either measured by the equivalent variation or the variation in surplus, is larger than the welfare obtained when focusing on the input market ($\Delta S^{output} > \Delta S^{input}$). This bias is caused by the negative externality that pushes the price in this market down (below the true social cost), generating a provision of this good above what is socially desirable. However, when detracting this social cost from the final demand for this good, both the EV and surplus approach converge to the value reported by ΔY_Z. Alternatively, the social cost can also be internalized by levying a tax on the consumption of this good in the model; causing that ΔY_Z equates to the surplus without *ex-post* adjustments.

A CGE model can also shed light on the welfare impact in cases of derived demand. As theoretically proven and tested in the models, the economic impact in the input market, in multiplicative terms ($\widetilde{P_Z}\widetilde{Y_Z}$), (input market multiplicative effect, Pme) coincides with the total welfare effect ($\widetilde{EV}$ and $\widetilde{M}$, which denote the equivalent variation and income variation in multiplicative terms) in all economic situations, except in open economy situations with a surplus or deficit, such as: $PME = \widetilde{M} = \widetilde{EV} = \widetilde{P_Z}\widetilde{Y_Z}$.

Moreover, it should be noted that CGE provides a comprehensive methodology to conduct counterfactual scenarios. Thus, once the model is built, it is relatively straightforward to conduct any policy analysis. Likewise, the *model closure* represents an issue of concern when conducting welfare analyses in CGE. Specifically, as demonstrated for the government's budgetary position in the final section, the way the government decides to finance a public investment does affect the welfare measurement. Fortunately, a CGE framework can address different *model closures*. Table 25 summarizes the main welfare measures and their divergences under different market situations in CGE.

Hence, we conclude that a project's net welfare effect conducted with CGE should equate to that of CBA when both methods are consistently applied. The presence of distortions or appraisal in an open economy, should not be a cause of divergence in the measurement of a project's net welfare effect, as CBA incorporates any relevant distortion in other related markets. In the case of the multiplier effect, when the value of the distortions is expected to be similar to the counterfactual, they can be ignored.

Table 25. Welfare measure divergences under different market situations in CGE

		SW^{CGE}	IWA	ΔCS	PME
Voluntary unemployment	**Closed economy**	Unbiased	$SW^{CGE} = IWA$	-*	-
	Open economy	Unbiased	$SW^{CGE} = IWA$	-	-
Involuntary unemployment	**Closed economy**	Unbiased	$SW^{CGE} = IWA$	-	-
	Open economy	Unbiased	$SW^{CGE} = IWA$	-	-
Derived demand	**Closed economy**	Unbiased	$SW^{CGE} = IWA$	$\Delta S^{output} = \Delta S^{input}$	$PME = \widehat{EV}$
	Open economy	Unbiased	$SW^{CGE} = IWA$	$\Delta S^{output} \neq \Delta S^{input}$	$PME \neq \widehat{EV}$
Derived demand and involuntary unemployment	**Closed economy**	Unbiased	$SW^{CGE} = IWA$	$\Delta S^{output} \neq \Delta S^{input}$	$PME = \widehat{EV}$
Derived demand and externalities	**Closed economy**	Biased**	$SW^{CGE} = IWA$	Unbiased*** $\Delta S^{output} \neq \Delta S^{input}$	$PME \neq \widehat{EV}$
Derived demand and imperfect competition	**Closed economy**	Unbiased	$SW^{CGE} = IWA$	$\Delta S^{output} \neq \Delta S^{input}$	$PME = \widehat{EV}$
Derived demand and different *model closure*	**Closed economy**	Unbiased	$SW^{CGE} = IWA$	$\Delta S^{output} \neq \Delta S^{input}$	$PME = \widehat{EV}$

*Not applicable

**Unbiased when endogenizing the externality's cost by levying a tax on the consumption of the good that is causing it.

***Biased when the externality is caused by a sector whose production is demanded from Y_Z.

References

Boardman, A. E., Greenberg, D. H., Vining, A. R., and Weimer, D. L. (2018). Cost-benefit analysis: concepts and practice. Cambridge University Press.

Böhringer, C., Rutherford, T.F. and Wiegard, W. (2003) Computable General Equilibrium Analysis: Opening a Black Box, *ZEW Discussion Paper* No. 03-56, Mannheim, Germany.

de Rus, G. (2021): Introduction to Cost-Benefit Analysis: Looking for Reasonable Shortcuts. 2nd edition. Edward Elgar. Cheltenham.

Deaton, A., and Muellbauer, J. (1980). Economics and Consumer Behavior. Cambridge University Press.

European Investment Bank (2013) The Economic Appraisal of Transport Projects at the EIB. Luxembourg: European Investment Bank.

Gilbert, J., and Tower, E. (2013). An introduction to numerical simulation for trade theory and policy. Singapore: World Scientific.

Hosoe, N., Gasawa, K. and Hashimoto, H. (2010). Textbook of Computable General Equilibrium Modelling: Programming and Simulations, London: Palgrave Macmillan.

Inchausti-Sintes, F., and Njoya, E. T. (2022). An overview of computable General Equilibrium Models. C-Bridge project. EIB Institute.

Johansson, P. O. (1993). Cost-benefit analysis of environmental change. Cambridge: Cambridge University Press.

Johansson, P. O. (2022). On the evaluation of large projects in closed and open economies. C-Bridge project. EIB Institute.

Johansson, P. O., and Kriström, B. (2016). Cost-benefit analysis for project appraisal. Cambridge, Ma: Cambridge University Press.

Johansson, P. O., and Kriström, B. (2022). On the social opportunity cost of unemployment. *Journal of Economic Policy Reform*, 25(3), 229-239.

Liu, W. F., and Turnovsky, S. J. (2005). Consumption externalities, production externalities, and long-run macroeconomic efficiency. *Journal of Public Economics,* 89(5-6), 1097-1129.

Lucas, R. E. Jr. (1988). On the mechanics of development planning. *Journal of Monetary Economics,* 22, 1, 3-42.

Mas-Colell, A., Whinston, J. and Green, J. (1995). Microeconomic Theory. Oxford University Press, New York.

Romer, P.M. (1986). Increasing return and long run growth. *Journal of Political Economy* 94, 1002–1037.

Rutherford, T. F. (1999). Applied general equilibrium modeling with MPSGE as a GAMS subsystem: An overview of the modeling framework and syntax. *Computational Economics*, 14(1-2), 1-46.

Varian, H. R. (1992). Microeconomic analysis (Vol. 2). New York: Norton.

Willig, R. (1976). Consumer's surplus without apology, *American Economic Review,* 66 (4), 589-97.

Wing, I. S. (2004). Computable general equilibrium models and their use in economy-wide policy analysis. *Technical Note, Joint Program on the Science and Policy of Global Change, MIT.*

ANNEX I

Proposition: *A rise in productivity enhances a welfare improvement*

Proof:

Let's assume the introduction of a productivity improvement technology in a closed economy with two factors of production K and L and one single good Y. And where superindex 0 and f denote the initial and final situation following introduction of the technology.

The first derivatives of the production are all positives:

$$\frac{\partial F^0}{\partial L} > 0, \frac{\partial F^0}{\partial K} > 0, \frac{\partial F^f}{\partial L} > 0, \frac{\partial F^f}{\partial K} > 0$$

But decreasing:

$\frac{\partial F^{0\prime}}{\partial L \partial L} < 0, \frac{\partial F^{0\prime}}{\partial K \partial K} < 0, \frac{\partial F^{f\prime}}{\partial L \partial L} < 0, \frac{\partial F^{f\prime}}{\partial K \partial K} < 0$

Finally, $\frac{\partial F^0}{\partial L} < \frac{\partial F^f}{\partial L}$ and $\frac{\partial F^0}{\partial K} < \frac{\partial F^f}{\partial K}$ capture the productivity improvement of the new technology.

By the circular flow of income, $Y = F(K, L) = M = wL + rK$.

Imposing that, the real price of L (w) and K (r) equals its marginal productivity:

$MP_L = \frac{\partial F}{\partial L} = \frac{w}{P}; MP_K = \frac{\partial F}{\partial K} = \frac{r}{P}$

By the Euler theorem and distinguishing between both situations:

$$Y^0 = F^0(K, L) = \frac{\partial F^0}{\partial L} L_0 + \frac{\partial F^0}{\partial K} K_0$$

$$Y^f = F^f(K, L) = \frac{\partial F^f}{\partial L} L_0 + \frac{\partial F^f}{\partial K} K_0$$

Remembering that $\frac{\partial F^0}{\partial L} < \frac{\partial F^f}{\partial L}$ and $\frac{\partial F^0}{\partial K} < \frac{\partial F^f}{\partial K}$, implies that:

$\frac{w_0}{P_0} < \frac{w_f}{P_f}$ and $\frac{r_0}{P_0} < \frac{r_f}{P_f}$

which also causes that: $Y^f > Y^0$ and that $M^f > M^0$. Hence, $EV = M^f - M^0 > 0$.